This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at http://books.google.com/

EDUCATIONAL BOOKS.

PUBLISHED BY MESSRS. BELL AND DALDY,

186, FLEET STREET, LONDON.

BIBLIOTHECA CLASSICA.

A Series of Greek and Latin Authors. With English Notes. 8vo. Edited by various Scholars, under the direction of G. Long, Esq. M.A. Classical Lecturer of Brighton College: and the late Rev. A. J. Macleane, M.A. Head Master of King Edward's School, Bath.

ÆSCHYLUS. By F. A. Paley, M.A. 18*s.*

CICERO'S ORATIONS. Edited by G. Long, M.A. 4 vols. Vol. I. 16*s.*; Vol. II. 14*s.*; Vol. III. 16*s.*; Vol. IV. 18*s.*

DEMOSTHENES. By R. Whiston, M.A. Head Master of Rochester Grammar School. Vol. I. 16*s.* Vol. II. *preparing.*

EURIPIDES. By F. A. Paley, M.A. 3 Vols. 16*s.* each.

HERODOTUS. By J. W. Blakesley, B.D. late Fellow and Tutor of Trinity College, Cambridge. 2 vols. 32*s.*

HESIOD. By F. A. Paley, M.A. 10*s.* 6*d.*

HORACE. By A. J. Macleane, M.A. 18*s.*

JUVENAL AND PERSIUS. By A. J. Macleane, M.A. 14*s.*

SOPHOCLES. By F. H. Blaydes, M.A. Vol. I. 18*s.*

TERENCE. By E. St. J. Parry, M.A. 18*s.*

VIRGIL. By J. Conington, M.A. Professor of Latin at Oxford. Vol. I. containing the Bucolics and Georgics. 12*s.* Vol. II. *in the press.*

PLATO. By W. H. Thompson, M.A. Vol. I. *preparing.*

FOREIGN CLASSICS.

With English Notes for Schools. Fcap. 8vo.

AVENTURES DE TÉLÉMAQUE, par Fénelon. Edited by C. J. Delille. *Second Edition, revised.* 4*s.* 6*d.*

HISTOIRE DE CHARLES XII. par Voltaire. Edited by L. Direy. *Second Edition, revised.* 3*s.* 6*d.*

SELECT FABLES OF LA FONTAINE. *Second Edition, revised.* Edited by F. Gasc, M.A. 3*s.*

PICCIOLA, by X. B. Saintine. Edited by Dr. Dubuc. 3*s.* 6*d.*

THE WALLENSTEIN OF SCHILLER, with Notes by Dr. A. Buchheim. [*Immediately.*

GRAMMAR-SCHOOL CLASSICS

With English Notes for Schools. Fcap. 8vo.

J. CÆSARIS COMMENTARII DE BELLO GALLICO. *Second Edition.* By G. LONG, M.A. *5s. 6d.*

CÆSAR DE BELLO GALLICO, Books 1 to 3. With English Notes for Junior Classes. By G. LONG, M.A. *2s. 6d.*

M. TULLII CICERONIS CATO MAJOR, SIVE DE SENECTUTE, LÆLIUS, SIVE DE AMICITIA, ET EPISTOLÆ SELECTÆ. By G. LONG, M.A. *4s. 6d.*

QUINTI HORATII FLACCI OPERA OMNIA. By A. J. MACLEANE, M.A. *6s. 6d.*

P. OVIDII NASONIS FASTORUM LIBRI SEX. By F. A. PALEY. *5s.*

C. SALLUSTII CRISPI CATILINA ET JUGURTHA. By G. LONG, M.A. *5s.*

TACITI GERMANIA ET AGRICOLA. By P. FROST, M.A. *3s. 6d.*

XENOPHONTIS ANABASIS, with Introduction. By J. F. MACMICHAEL, B.A. *New Edition.* *5s.*

XENOPHONTIS CYROPÆDIA. By G. M. GORHAM, M.A. late Fellow of Trinity College, Cambridge. *6s.*

Uniform with the above.

THE NEW TESTAMENT IN GREEK. With English Notes and Prefaces by J. F. MACMICHAEL, B.A. 730 pages. *7s. 6d.*

CLASSICAL TABLES. 8vo.

NOTABILIA QUÆDAM: or, the Principal Tenses of such Irregular Greek Verbs and such elementary Greek, Latin, and French Constructions as are of constant occurrence. *1s. 6d.*

IRREGULAR GREEK VERBS. *1s.*

GREEK ACCIDENCE. By the Rev. P. FROST, M.A. *1s.*

LATIN ACCIDENCE. By the Rev. P. FROST, M.A. *1s.*

LATIN VERSIFICATION. *1s.*

THE PRINCIPLES OF LATIN SYNTAX. *1s.*

HOMERIC DIALECT; its Leading Forms and Peculiarities. By J. S. BAIRD, T.C.D. *1s. 6d.*

A CATALOGUE OF GREEK VERBS, IRREGULAR and Defective. By J. S. BAIRD, T.C.D. *New Edition, revised.* *3s. 6d.*

RICHMOND RULES TO FORM THE OVIDIAN DISTICH, &c. By J. TATE, M.A. *New Edition, revised.* *1s. 6d.*

BELL AND DALDY, 186, FLEET STREET, LONDON.

FACTS AND FORMULÆ

IN

PURE MATHEMATICS

AND

NATURAL PHILOSOPHY.

LONDON:
R. CLAY, SON, AND TAYLOR, PRINTERS,
BREAD STREET HILL.

A COMPENDIUM OF

FACTS AND FORMULÆ

IN

PURE MATHEMATICS AND NATURAL PHILOSOPHY.

BODLEIANA BIBLIOTHECA

BY G. R. SMALLEY, B.A., F.R.A.S.
OF ST. JOHN'S COLLEGE, CAMBRIDGE;
LECTURER ON NATURAL PHILOSOPHY IN KING'S COLLEGE, LONDON; LATE HEAD MATHEMATICAL MASTER IN KING'S COLLEGE SCHOOL.

LONDON:
BELL AND DALDY, 186, FLEET STREET.
1862.

~~180. e. 100.~~
187. c. 34.

BIBLIOTHECA · BODLEIANA ·
DOMINUS ILLUMINATIO MEA

PREFACE.

THE present work is designed as a book of reference for the Mathematical student, and to supply a want much felt in the lecture-room by those who require a hand-book of established Principles and Formulæ to enable them to follow the explanations of the lecturer.

It is not intended for the gratification of the superficial learner, or to assist those who are content to stake their chance of success in a competitive examination upon the precarious and obnoxious system of cramming; but it may be used with advantage by the student to test his own proficiency in writing out the proofs of the formulæ and fundamental propositions, and may often save the tutor himself much time and trouble in the preparation of a book-work examination.

Many years' experience in Mathematical tuition, as well as in the lecture-room, have convinced the

compiler of the utility of such a compendium. It has been his endeavour to render the present work sufficiently complete, and to keep in view a point too much lost sight of in Mathematical works—uniformity and simplicity of Notation; and, whilst every care has been taken to secure accuracy and perspicuity of arrangement, no merit is claimed for it beyond that which is due to the labour bestowed upon its preparation.

KING'S COLLEGE, LONDON,
April, 1862.

INDEX.

ARITHMETIC.

ALGEBRA.

TRIGONOMETRY.

SPHERICAL TRIGONOMETRY.

ANALYTICAL GEOMETRY AND CONIC SECTIONS.

MENSURATION.

DIFFERENTIAL CALCULUS.

INTEGRAL CALCULUS.

GEOMETRY OF THREE DIMENSIONS.

NATURAL PHILOSOPHY.

STATICS.

DYNAMICS.

HYDROSTATICS.

OPTICS.

ASTRONOMY.

HEAT.

ACOUSTICS.

APPENDIX.

Abridged Tables.

ERRATA.

Page 14, art. 19, *for* "cos," *read* "cosec."
— 31, — 69, *for* "$x\,b$," *read* "$x\,y$."
— 47, — 3, *for* "Except $n = 1$," *read* "Except $n = -1$."
— 49, — 31, *for* "x_m," *read* "x^m."
— 53, — 16, *for* "two parallelisms," *read* "parallelism."
— 55, — 24, *for* "$(z - c)^2$," *read* "$(z - l)^2$."
— 55, — 28, *for* "semi-axis," &c. *read* "the semi-axes of the generating ellipse being a and b."
— 79, — 86, *for* "Abscissa," *read* "Abscissæ."

ARITHMETIC.

CRITERIA OF DIVISIBILITY.

I. A NUMBER is divisible by 2, 4, or 8, when the number formed by the last one, two, or three figures on the right, is divisible by 2, 4, or 8 respectively.

II. Every number ending with 5 or 0, is divisible by 5 or 10 respectively.

III. Every number is divisible by 3 or 9 when the sum of the digits is divisible by 3 or 9 respectively.

IV. Every number is divisible by 11 when the difference between the sum of the figures in the odd places and the sum of those in the even places is divisible by 11.

V. Every compound number will divide a given quantity when its component factors will divide that quantity.

ALGEBRA.

ALGEBRAICAL SYMBOLS.

$=$	denotes,	equal to, or equals.
$+$	„	plus, or together with.
$-$	„	minus, or less than.
$\times$	„	multiplied into.

$\div$	denotes,	divided by.
$\sim$	„	difference between.
$>$	„	greater than.
$<$	„	less than.
$\propto$	„	varies as.
∞	„	infinity.
$\lfloor n$	„	factorial n, or the product of the natural numbers from 1 to n.
a^n	„	$a \times a \times a \ldots$ to n factors.
$\sqrt[n]{a}$ or $a^{\frac{1}{n}}$	„	the n^{th} root of a.
$a^{\frac{m}{n}}$	„	the n^{th} root of a^m.

Anomalous Results.

$$a \times 0 = 0.$$

$$\frac{0}{a} = 0.$$

$$\frac{a}{0} = \infty.$$

$$\frac{0}{0} = \text{an indeterminate quantity.}$$

FORMULÆ.

Laws of Indices.

1. $a^m \times a^n = a^{m+n}$. I.

2. $a^m \div a^n = a^{m-n}$; II.

   or $\dfrac{a^m}{a^n} = a^{m-n}$.

3. $(a^m)^n = a^{mn}$. III.

4. $a^0 = 1$.

5. $$a^{-n} = \frac{1}{a^n}.$$

6. $$a^m\, b^m = (a\,b)^m.$$

7. $$\sqrt[m]{a}\,.\,\sqrt[m]{b} = \sqrt[m]{a\,b}.$$

8. $$a^2 - b^2 = (a + b)\,(a - b).$$

9. $$(a \pm b)^2 = a^2 \pm 2\,a\,b + b^2.$$

10. $$(a + b + c)^2 = a^2 + b^2 + c^2 + 2\,a\,b + 2\,a\,c + 2\,b\,c.$$

11. $$(a \pm b)^3 = a^3 \pm 3\,a^2 b + 3\,a\,b^2 \pm b^3.$$

12. $$(a + b + c)^3 = a^3 + b^3 + c^3 + 3\,(a^2 b + a\,b^2 + a^2 c + a\,c^2 + b^2 c + b\,c^2) + 6\,a\,b\,c.$$

13. $$a^3 \pm b^3 = (a \pm b)\,(a^2 \mp a\,b + b^2).$$

14. $$\frac{a^3 \pm b^3}{a \pm b} = a^2 \mp a\,b + b^2.$$

QUADRATIC EQUATIONS.

General Form.

15. $$x^2 + p\,x + q = 0.$$

General Solution.

16. $$x = \tfrac{1}{2}\,(-p \pm \sqrt{p^2 - 4\,q}).$$

roots equal if . . . . $p^2 = 4\,q.$
roots impossible if . . . . . . $p^2 < 4\,q.$
roots possible and unequal if $p^2 > 4\,q.$

General Properties.

17. $-p$ = sum of the roots.
q = product of the roots.

RATIO; PROPORTION AND VARIATION.

$a : b$ denotes the ratio of a to b.

$\frac{a}{b}$ " measure of $a : b$.

$a : b = c : d$ or $\frac{a}{b} = \frac{c}{d}$ denotes the equality of the ratios $a : b$ and $c : d$.

18. If $a : b = c : d$ or $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$.

19. Invertendo

$$b : a = d : c, \text{ or } \frac{b}{a} = \frac{d}{c}.$$

20. Alternando

$$a : c = b : d, \text{ or } \frac{a}{c} = \frac{b}{d}.$$

21. Componendo

$$a + b : b = c + d : d, \text{ or } \frac{a+b}{b} = \frac{c+d}{d}.$$

22. Dividendo

$$a - b : b = c - d : d, \text{ or } \frac{a-b}{b} = \frac{c-d}{d}.$$

23. Convertendo

$$a : a - b = c : c - d, \text{ or } \frac{a}{a-b} = \frac{c}{c-d}.$$

24. $a \pm b : a \mp b = c \pm d : c \mp d$, or $\frac{a \pm b}{a \mp b} = \frac{c \pm d}{c \mp d}$.

25. If $a : b = b : c$, then $\frac{a}{c} = \frac{a^2}{b^2}$,

or $a : c$ in the duplicate ratio of $a : b$.

26. If $a : b = b : c = c : d$, then $\frac{a}{d} = \frac{a^3}{b^3}$,

or $a : d$ in the triplicate ratio of $a : b$.

27. If $a : b = b : c = c : d = d : e = e : f$. &c. . . .

$$\frac{a + c + e + \ldots}{b + d + f + \ldots} = \frac{a}{b},$$

or sum of antecedents is to the sum of the consequents as one of the antecedents is to its consequent.

28. If $V \propto v$,
$V = Cv$; C being a constant quantity.

29. If $V \propto$ inversely as v,
$$V = \frac{C}{v}.$$

30. If $V \propto v_1$ and $V \propto v_2$,
$V \propto v_1 \pm v_2$;
$V \propto \sqrt{v_1 v_2}$.

31. If $V \propto v_1$ when v_2, v_3 . . . are constant,
" $V \propto v_2$ " v_1 v_3 . . . "
" $V \propto v_3$ " v_1 v_2 . . . "
then $V \propto$ product when all vary,
or
$$V = C \cdot v_1 v_2 v_3 \ldots$$

PROGRESSIONS.

a = first term.
l = last.
d = common difference.
r = common ratio.
n = number of terms.
s = sum of n terms.

Arithmetical Progression (A.P.).

32. n^{th} term $= a + \overline{n-1} \cdot d$.

33. $$s . = (a + l)\frac{n}{2} = (2a + \overline{n-1} \cdot d)\frac{n}{2}$$

34. Arithmetic mean = semi-sum of the extremes.

Geometrical Progression (G.P.).

35. n^{th} term $= ar^{n-1}$.

36. $$s = a \cdot \frac{r^n - 1}{r - 1}.$$

37. Σ = limit of the sum in inf.

$$= \frac{a}{1-r}.$$

38. Geometric mean $= \sqrt{\text{product of the extremes}}$.

Harmonical Progression (H.P.).

Def. $a, b, c, d, \ldots$ are in H. P. if

$$a : c = a - b : b - c,$$
$$b : d = b - c : c - d,$$
&c. . . .

39. The reciprocals of quantities in H.P. are in A.P.

or $\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b} = \ldots$

40. n^{th} term $= \dfrac{ab}{(n-1)a - (n-2)b}$.

41. Harmonic mean between x and y

$$= \frac{2xy}{x+y}$$
$$= \frac{(\text{Geom. mean})^2}{\text{Arith. mean}}.$$

Piles of Balls.

n = number of balls in a side of the base.
= number of courses.

42. Number of balls in a triangular pile

$$= \frac{n}{6}(n+1)(n+2).$$

43. Number of balls in a square pile

$$= 1^2 + 2^2 + 3^2 + \ldots + n^2,$$

$$= \frac{n}{6}(n + 1)(2n + 1).$$

44. Number of balls in a rectangular pile

$$= \frac{n}{6}(n + 1)(3l - n + 1).$$

(l and n being the number of balls in the longer and shorter sides of base.)

VARIATIONS, PERMUTATIONS, AND COMBINATIONS.

45. Number of variations of n things taken r together $= n(n - 1)(n - 2) \ldots (n - r + 1)$.

46. Number of permutations of n things $= n(n - 1)(n - 2) \ldots 3 . 2 . 1$.

47. Number of permutations of n things when the quantities recur, $p, q, r \ldots$ times

$$= \frac{1 . 2 . 3 \ldots n}{1 . 2 . 3 \ldots p . 1 . 2 . 3 \ldots q 1 . 2 . 3 \ldots r \ldots}.$$

$$= \frac{\underline{|n}}{\underline{|p} \cdot \underline{|q} \cdot \underline{|r} \ldots}.$$

48. Number of combinations of n things taken r together

$$= \frac{n(n - 2)(n - 1) \ldots (n - r + 1)}{1 . 2 . 3 \ldots r}.$$

49. Number of combinations of n sets of things containing $p, q, r \ldots$ things in each set respectively $= p, q, r \ldots$

50. Total number of combinations of n things $= 2^n - 1$.

BINOMIAL THEOREM.

51. $(a + b)^n = a^n + na^{n-1} b + n . \frac{n-1}{2} a^{n-2} b^2 + n . \frac{n-1}{2} . \frac{n-2}{3} a^{n-3} b^3 + \ldots$

52. r^{th} term $= n \cdot \dfrac{n-1}{2} \cdot \dfrac{n-2}{3} \cdots$

$\dfrac{n-r+2}{r-1} \cdot a^{n-r+1} b^{r-1}$.

53. Greatest term $= r^{\text{th}}$ term

if r be the first whole number $> (n+1)\dfrac{b}{a+b}$.

54. Number of terms $= n+1$

if $n =$ integer.

55. Sum of the coefficients $= 2^n$.

PROPERTIES OF NUMBERS.

56. The product of r consecutive numbers is divisible by $1 \cdot 2 \cdot 3 \ldots r$.

57. General form of a square number is $5n$, or $5n \pm 1$.

58. Fermat's theorem,

$$\frac{N^{n-1} - 1}{n} = \text{integer},$$

n being a prime number, and N prime to n.

59. Wilson's theorem,

$$\frac{1 \cdot 2 \cdot 3 \ldots \overline{n-1} + 1}{n} = \text{integer},$$

n being a prime number.

INTEREST, DISCOUNT, AND ANNUITIES.

Notation.

P denotes principal in pounds.
r „ rate of interest, or interest on £1 for 1 year.
n „ number of years.
I „ interest on P for n years.
M „ amount.
V „ present value.
D „ discount.
A „ annuity.

60. Simple interest,

$$I = Pnr,$$
$$M = P(1 + nr).$$

61. Compound interest,

$$I = M - P,$$
$$M = P(1 + r)^n,$$
$$V = \frac{P}{(1 + r)^n}.$$

62. Discount,

$$D = \frac{Pnr}{1 + nr}.$$
$$V = P - D = \frac{P}{1 + nr}.$$

63. Terminable annuities,

$$M = A \cdot \frac{(1 + r)^n - 1}{r}.$$
$$V = \frac{A}{r}\left\{1 - \frac{1}{(1 + r)^n}\right\}.$$

64. Perpetual annuities,

$$V = \frac{A}{r}.$$

65. In an annuity deferred for d years, and to continue for n years,

$$V = \frac{A}{r}\left\{(1 + r)^{-d} - (1 + r)^{-d-n}\right\}.$$

PROBABILITIES.

If a = number of ways in which an event may happen,
b = number of ways in which an event may fail.

66. Probability of the event happening

$$= \frac{a}{a + b}.$$

67. Probability of the event failing

$$= \frac{b}{a+b}.$$

68. Probability of an event's happening r times exactly in n trials

$$= \frac{n\,\frac{n-1}{2}\cdot\frac{n-2}{3}\cdots\frac{n-r+1}{r}\,a^r\,b^{n-r}}{(a+b)^n}.$$

69. Probability that any number of independent events will all happen

$$= \frac{1}{p_1 p_2 p_3 \ldots}; \text{ where } \frac{1}{p_1}, \frac{1}{p_2}, \frac{1}{p_3} \ldots$$

represent the probabilities that each will happen.

ASSURANCES.

70. Single premium for an assurance of £1 on life aged m

$$= \frac{1 - r\,.\,v_m}{1+r}.$$

71. Annual premium

$$= \frac{\text{single premium}}{1 + v_m};$$

where v_m = present value of £1 per annum on a life aged m years.

LOGARITHMS.

Def. If $a^l = N$, l = logarithm of N to base a . or $l = \log_a N$.

Common base = 10.

Napierian base = e (= 2·7182818 nearly)

$\log_a \infty = \infty$, and $\log_a 1 = 0$, and $\log_a 0 = -\infty$.

Characteristic—the integral part of a logarithm.

Mantissa—the decimal part of a logarithm.

If n be the number of digits in the integral part of a quantity N greater than 1, or the number of cyphers immediately after the decimal point in a quantity N less than 1, the characteristic of $\log_{10} N$ is $n - 1$, or $-(n+1)$, respectively.

Fundamental Properties of Logarithms.

72. $\log_a N_1 . N_2 . N_3 \ldots = \log_a N_1 + \log_a N_2 + \log_a N_3 + \ldots$

73. $$\log_a \frac{N}{D} = \log_a N - \log_a D.$$

74. $$\log_a N^n = n \log_a N.$$

75. $$\log_a \sqrt[n]{N} = \frac{\log_a N}{n}.$$

76. $$\log_a N = \log_a b . \log_b N.$$

Modulus of a System of Logarithms—the constant multiplier which connects two systems.

77. Modulus of the common system $= \dfrac{1}{\log_e 10}$.

Properties of Logarithms to base 10.

78. The characteristic may be determined by inspection.

79. The mantissa is the same for the logarithms of all numbers having the same *significant* digits.

Principle of Proportional Parts.

80. If $N + x$ be a quantity lying between the consecutive numbers N and $N + 1$, N being large and x small, l, $l + y$, $l + D$ their corresponding logarithms; $x : 1 = y : D$, or $x = \dfrac{y}{D}$ and $y = Dx$, that is, for large numbers and small differences, the increments of the numbers are proportional to the increments of their logarithms.

Exponential Theorems.

81. $$a^x = 1 + \log_e a \,.\, x + (\log_e a)^2 \,.\, \frac{x^2}{1\,.\,2} + (\log_e a)^3 \,.\, \frac{x^3}{1\,.\,2\,.\,3} + \ldots$$

82. $$e^x = 1 + x + \frac{x^2}{1\,.\,2} + \frac{x^3}{1\,.\,2\,.\,3} + \ldots$$

Logarithmic Series.

83. $$\log_a (1 + x) = \frac{1}{\log_e a}\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots\right).$$

84. $$\log_e (1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

Formulæ for Computation of Logarithms to base 10.

85. $$\log (n + 1) = \log n + \frac{2}{\log_e 10}\left(\frac{1}{2n + 1} + \frac{1}{3\,(2n + 1)^3} + \frac{1}{5\,(2n + 1)^5} + \ldots\right).$$

86. $$\log (n + 1) = 2 \log n - \log (n - 1) - \frac{2}{\log_e 10}\left(\frac{1}{2n^2 - 1} + \frac{1}{3\,(2n^2 - 1)^3} + \ldots\right).$$

87. $$\frac{1}{\log_e 10} = \cdot 434294482 \text{ approximately.}$$

TRIGONOMETRY.

In *any* right-angled triangle, the angle A is contained by the *base* and *hypothenuse*, and subtended by the *perpendicular*.

TRIGONOMETRICAL RATIOS AND THEIR RECIPROCALS.

1. $$\text{sine } A = \frac{\text{perp}^{\text{r}}}{\text{hyp}^{\text{s}}}.$$

2. $$\frac{1}{\text{sine } A} = \frac{\text{hyp}^{\text{s}}}{\text{perp}^{\text{r}}} = \text{cosecant } A.$$

3. $$\text{cosine } A = \frac{\text{base}}{\text{hyp}^{\text{s}}}.$$

4. $$\frac{1}{\text{cosine } A} = \frac{\text{hyp}^{\text{s}}}{\text{base}} = \text{secant } A.$$

5. $$\text{tangent } A = \frac{\text{perp}^{\text{r}}}{\text{base}}.$$

6. $$\frac{1}{\text{tangent } A} = \frac{\text{base}}{\text{perp}^{\text{r}}} = \text{cotangent } A.$$

$$\text{Versed sine } A = 1 - \text{cosine } A.$$

7. chord A, of circle radius r, $= 2r \sin \dfrac{A}{2}$.

8. $$\tan A = \frac{\sin A}{\cos A}. \cot A = \frac{\cos A}{\sin A}.$$

MEASUREMENT OF ANGLES.

9. Unit of English measure (degree):

$$1^\circ = \frac{1}{90}^{\text{th}} \text{ of right } \angle.$$

$$1^\circ = 60', \quad 1' = 60''.$$

10. Unit of French measure (grade):

$$1^g = \frac{1}{100}^{\text{th}} \text{ of right } \angle.$$

$$1^g = 100', \quad 1' = 100''.$$

11. Circular measure is the numerical ratio of arc to radius. Unit of circular measure is the angle whose arc = radius.

If π represent the circular measure of two right angles,
D = number of degrees in any angle A,
G = " grades "
θ = circular measure of "

12. $$D = G - \frac{G}{10}.$$

$$G = D + \frac{D}{9}.$$

13. $$\theta = \frac{D}{180} \cdot \pi, \quad D = \frac{180}{\pi} \cdot \theta.$$

14. Numerical value of $\pi = 3{\cdot}14159$, or $3\frac{1}{7}$ nearly.

" $\frac{180}{\pi} = 57^\circ{\cdot}29578$ "

15. Limiting value of $\frac{\sin\theta}{\theta}$ and $\frac{\tan\theta}{\theta} = 1$.

16. For *very small* angles, $\sin\theta = \theta = \tan\theta$.

17. Complement of $A^\circ = 90^\circ - A^\circ$.

18. $\sin(90 - A) = \cos A$; $\cos(90 - A) = \sin A$.

19. $\cos(90 - A) = \sec A$; $\sec(90 - A) = \operatorname{cosec} A$.

20. $\tan(90 - A) = \cot A$; $\cot(90 - A) = \tan A$.

21. Supplement of $A^\circ = 180^\circ - A^\circ$.

22. $\sin(180 - A) = \sin A$.

23. $\cos(180 - A) = -\cos A$.

24. $\tan(180 - A) = -\tan A$.

25. $\sin(180 + A) = -\sin A$.

26. $\cos(180 + A) = -\cos A$.

27. $\tan(180 + A) = \tan A$.

General Forms.

28. $\sin\theta = \sin\{n\pi + (-1)^n\theta\}$.

29. $\cos\theta = \cos\{2n\pi \pm \theta\}$.

30. $\tan\theta = \tan\{n\pi + \theta\}$.

n being *any* positive or negative integer, or zero.

31. *Changes of the Trigonometrical Ratios through the four Quadrants.*

$\angle A$	$\sin A$	$\cos A$	$\tan A$	$\cot A$	$\sec A$	$\operatorname{cosec} A$
0° to 90°	+ 0 to 1	+ 1 to 0	+ 0 to ∞	+ ∞ to 0	+ 1 to ∞	+ ∞ to 1
90 to 180	+ 1 to 0	— 0 to 1	— ∞ to 0	— 0 to ∞	— ∞ to 1	+ 1 to ∞
180 to 270	— 0 to 1	— 1 to 0	+ 0 to ∞	+ ∞ to 0	— 1 to ∞	— ∞ to 1
270 to 360	— 1 to 0	+ 0 to 1	— ∞ to 0	— 0 to ∞	+ ∞ to 1	— 1 to ∞

Negative Angles.

32. $\sin(-A) = -\sin A$.

33. $\cos(-A) = \cos A$.

34. $\tan(-A) = -\tan A$.

Common Formulæ.

35. $\sin^2 A + \cos^2 A = 1$.

36. $\sin A = \sqrt{1 - \cos^2 A}$.

37. $\cos A = \sqrt{1 - \sin^2 A}$.

38. $$\sec A = \sqrt{1 + \tan^2 A}.$$

39. $$\operatorname{cosec} A = \sqrt{1 + \cot^2 A}.$$

40. $$\sin A = \frac{\tan A}{\sqrt{1 + \tan^2 A}}.$$

41. $$\cos A = \frac{1}{\sqrt{1 + \tan^2 A}}.$$

42. $$\sin (A + B) = \sin A \cos B + \cos A \sin B.$$

43. $$\sin (A - B) = \sin A \cos B - \cos A \sin B.$$

44. $$\cos (A + B) = \cos A \cos B - \sin A \sin B.$$

45. $$\cos (A - B) = \cos A \cos B + \sin A \sin B.$$

46. $$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \,.\, \tan B}.$$

47. $$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \,.\, \tan B}.$$

48. $$\sin 2A = 2 \sin A \cos A.$$

49. $$\cos 2A = \cos^2 A - \sin^2 A.$$

50. $$\cos 2A = 2 \cos^2 A - 1.$$

51. $$\cos 2A = 1 - 2 \sin^2 A.$$

52. $$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}.$$

53. $$\sin 3A = 3 \sin A - 4 \sin^3 A.$$

54. $$\cos 3A = 4 \cos^3 A - 3 \cos A.$$

55. $$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}.$$

56. $$\sin A - \sin B = 2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}.$$

57. $$\cos A + \cos B = 2 \cos \frac{A + B}{2} \cos \frac{A - B}{2}.$$

58. $$\cos B - \cos A = 2 \sin \frac{A + B}{2} \sin \frac{A - B}{2}.$$

59. $\tan A + \tan B = \dfrac{\sin (A+B)}{\cos A \cos B}.$

60. $\tan A - \tan B = \dfrac{\sin (A-B)}{\cos A \cos B}.$

61. $\cot A + \cot B = \dfrac{\sin (A+B)}{\sin A \,.\, \sin B}.$

62. $\cot B - \cot A = \dfrac{\sin (A-B)}{\sin A \,.\, \sin B}.$

63. $$\sin \frac{A}{2} = \frac{1}{2}\left(\sqrt{1+\sin A} - \sqrt{1-\sin A}\right).$$

64. $$\cos \frac{A}{2} = \frac{1}{2}\left(\sqrt{1+\sin A} + \sqrt{1-\sin A}\right).$$

65. $$\tan \frac{A}{2} = \frac{1}{\tan A}\left(-1 + \sqrt{1+\tan^2 A}\right).$$

66. $\sin (A+B) \sin (A-B) = \sin^2 A - \sin^2 B.$

67. $\tan A + \tan B + \tan C = \tan A \,.\, \tan B \,.\, \tan C,$
(where $A + B + C = 180$).

68. *Numerical Values of certain Trigonometrical Ratios.*

$$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}.$$

$$\cos 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}.$$

$$\tan 30^\circ = \cot 60^\circ = \frac{1}{\sqrt{3}}.$$

$$\sec 30^\circ = \operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}.$$

$$\operatorname{cosec} 30^\circ = \sec 60^\circ = 2.$$

$$\cot 30^\circ = \tan 60^\circ = \sqrt{3}.$$

$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}.$$

$$\tan 45^\circ = \cot 45^\circ = 1.$$

$$\sec 45^\circ = \operatorname{cosec} 45^\circ = \sqrt{2}.$$

$$\sin 15^\circ = \cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}.$$

$$\cos 15^\circ = \sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}.$$

$$\tan 15^\circ = \cot 75^\circ = 2-\sqrt{3}.$$

$$\sin 18^\circ = \cos 72^\circ = \frac{\sqrt{5}-1}{4}.$$

$$\sin 54^\circ = \cos 36^\circ = \frac{\sqrt{5}+1}{4}.$$

SOLUTION OF TRIANGLES.

a, b, c denote the sides opposite the angles A, B, C.

$$s = \frac{a+b+c}{2}.$$

Δ = area of the triangle.
r = radius of the inscribed circle.
R = „ circumscribed circle.
r_a, r_b, r_c = radii of the escribed circles touching sides a, b, c respectively.

69. $$\sin A : \sin B : \sin C = a : b : c;$$

or $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

70. $$a^2 = b^2 + c^2 - 2bc\cos A;$$

or, $$\cos A = \frac{b^2+c^2-a^2}{2bc}.$$

71. $$a = (b - c)\frac{\cos\frac{A}{2}}{\cos\theta} \text{ when } \tan\theta = \frac{b+c}{b-c}\tan\frac{A}{2}.$$

72. $$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cdot\cot\frac{C}{2}.$$

73. $$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}.$$

74. $$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}.$$

75. $$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

76. $$\sin A = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}.$$

77. $$\Delta = \frac{1}{2}bc\sin A = \sqrt{s(s-a)(s-b)(s-c)}$$

78. $$c = a\cos B + b\cos A.$$

79. $$r = \frac{bc\sin A}{2s}.$$
$$= \frac{\Delta}{s}.$$
$$= \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

80. $$R = \frac{abc}{4\Delta}.$$
$$= \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}.$$

81. $$Rr = \frac{abc}{4s}.$$

82. $$r_a = \frac{\Delta}{s-a}.$$

83. $$r_b = \frac{\Delta}{s - b}.$$

84. $$r_c = \frac{\Delta}{s - c}.$$

85. Distance between the centres of the inscribed and circumscribed circles $= \sqrt{R^2 - 2Rr}$.

De Moivre's Theorem.

86. $(\cos\theta + \sqrt{-1}\sin\theta)^n = \cos n\theta + \sqrt{-1}\sin n\theta.$

87. If $\cos\theta + \sqrt{-1}\sin\theta = x.$

$$\cos\theta - \sqrt{-1}\sin\theta = \frac{1}{x}.$$

$$2\cos n\theta = x^n + \frac{1}{x^n}. \qquad (1)$$

$$2\sqrt{-1}\sin n\theta = x^n - \frac{1}{x^n}. \qquad (2)$$

88. *Euler's Series.*

$$\sin\theta = \theta - \frac{\theta^3}{\underline{|3}} + \frac{\theta^5}{\underline{|5}} - \frac{\theta^7}{\underline{|7}} \cdots$$

$$\cos\theta = 1 - \frac{\theta^2}{\underline{|2}} + \frac{\theta^4}{\underline{|4}} - \frac{\theta^6}{\underline{|6}}$$

$$\sin\theta = \frac{1}{2\sqrt{-1}}\left(e^{\theta\sqrt{-1}} - e^{-\theta\sqrt{-1}}\right).$$

$$\cos\theta = \frac{1}{2}\left(e^{\theta\sqrt{-1}} + e^{-\theta\sqrt{-1}}\right).$$

89. *Gregory's Series.*

$$\theta = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta - \ldots$$

90. *Machin's Series.*

$$\frac{\pi}{4} = 4\left(\frac{1}{5} - \frac{1}{3}\cdot\frac{1}{5^3} + \frac{1}{5}\cdot\frac{1}{5^5} - \cdots\right)$$
$$-\left\{\frac{1}{239} - \frac{1}{3}\left(\frac{1}{239}\right)^3 + \cdots\right)$$

SPHERICAL TRIGONOMETRY.

DEFINITIONS.

I. *Great Circle.*—The boundary of every section of a sphere made by a plane passing through the centre.

II. *Small Circle.*—The boundary of every section of a sphere made by a plane *not* passing through the centre.

III. *Poles of a Circle.*—The extremities of that diameter of the sphere which is perpendicular to the plane of the circle.

IV. *Spherical Triangle.*—The portion of the surface of a sphere contained by three arcs of great circles which cut one another two and two.

V. *Lune.*—The spherical surface included between two semicircles containing an angle θ.

FUNDAMENTAL FORMULÆ.

When, A, B, C denote the angles of a spherical triangle $\triangle$
a, b, c denote the sides respectively opposite.

$$s = \frac{a+b+c}{2} \text{ and } S = \frac{A+B+C}{2}.$$

$$E = A + B + C - 180 = \textit{spherical excess.}$$

1. $$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

2. $\sin A : \sin B : \sin C = \sin a : \sin b : \sin c.$

3. $$\cot a \sin b = \cot A \sin C + \cos b \cos C.$$

4. $$\cos A = -\cos B \cos C + \sin B \sin C \cos a.$$

Napier's Analogies.

5. $$\tan \frac{A+B}{2} = \frac{\cos \frac{a-b}{2}}{\cos \frac{a+b}{2}} \cdot \cot \frac{C}{2}.$$

6. $$\tan \frac{A-B}{2} = \frac{\sin \frac{a-b}{2}}{\sin \frac{a+b}{2}} \cdot \cot \frac{C}{2}.$$

7. $$\tan \frac{a+b}{2} = \frac{\cos \frac{A-B}{2}}{\cos \frac{A+B}{2}} \cdot \tan \frac{c}{2}.$$

8. $$\tan \frac{a-b}{2} = \frac{\sin \frac{A-B}{2}}{\sin \frac{A+B}{2}} \cdot \tan \frac{c}{2}.$$

Solution of Right-Angled Triangles.

$C = 90°$, c = hypothenuse.

Circular parts, a, b, $90 - c$, $90 - A$, $90 - B$.

9.

MIDDLE PART.	ADJACENT PART.	OPPOSITE PART.
a	b; $90 - B$	$90 - c$; $90 - A$
b	a; $90 - A$	$90 - c$; $90 - B$
$90 - c$	$90 - A$; $90 - B$	a; b
$90 - A$	b; $90 - c$	a; $90 - B$
$90 - B$	a; $90 - c$	b; $90 - A$

Napier's Rules.

10. Sine of the middle part = product of the tangents of the adjacent parts.

11. Sine of the middle part = product of the cosines of the opposite parts.

Solution of Oblique-Angled Triangles.

12. Given s,

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b\sin c}}.$$

13. $$\cos\frac{A}{2} = \sqrt{\frac{\sin s\,.\,\sin(s-a)}{\sin b\sin c}}.$$

14. Given S,

$$\sin\frac{a}{2} = \sqrt{\frac{-\cos S\cos(S-A)}{\sin B\sin C}}.$$

15. $$\cos\frac{a}{2} = \sqrt{\frac{\cos(S-B)\cos(S-C)}{\sin B\,.\,\sin C}}.$$

Other cases by common Formulæ and Napier's Analogies.

16. Area of lune $= \dfrac{\theta}{\pi}$. area of hemisphere.

17. Area of spherical triangle $= \dfrac{E}{360}$. area of hemisphere.

18. *Legendre's Theorem.* If A' B' C' are angles of a plane triangle whose sides are of the same length as those of a

corresponding spherical triangle, and small compared with the radius of the sphere; then—

$$A = A' + \frac{E}{3};\ B = B' + \frac{E}{3};\ C = C' + \frac{E}{3}.$$

ANALYTICAL GEOMETRY AND CONIC SECTIONS.

DEFINITIONS.

I. *Co-ordinate axes.*—Two straight lines intersecting each other in a point called *the origin*, to which the position of a point in the same plane with them is referred. The axes are *rectangular* or *oblique*, according as they intersect each other at *right angles* or *obliquely*.

II. *Ordinate of a point.*—The distance of the point from one of the axes, measured parallel to the other axis.

III. *Abscissa of a point.*—The part of the axis cut off by the ordinate.

IV. *Co-ordinates.*—The *abscissa* and *ordinate*.

V. *Radius vector of a point.*—Its distance from a *fixed* point called the *pole*.

VI. *Vectorial angle or angle of revolution.*—The angle which the *radius vector* makes with a *fixed* line called the *initial line.*

VII. *Polar co-ordinates of a point.*—The radius vector and vectorial angle.

VIII. *Equation to a line.*—The equation which expresses generally the relation between the co-ordinates of any point whatever in the line.

IX. *Locus of an equation.*—The line which contains all the points determined by the proposed equation.

FORMULÆ.

General Notation and Abbreviations.

x denotes the abscissa of a point.
y „ ordinate „
ρ „ radius vector „
θ „ vectorial angle „
$(x\ y)$ denotes the point whose co-ordinates are x and y.

FORMULÆ FOR TRANSFORMATION OF CO-ORDINATES.

When h denotes the abscissa of a new origin.
k „ ordinate of a new origin.
ω „ angle between two oblique axes.
a „ „ the axes of x.
β „ „ the axes of y.

1. For change of origin only,

$$x \text{ becomes } x + h,$$
$$y \text{ becomes } y + k.$$

2. For change of direction of axes, both systems rectangular,

$$x \text{ becomes } x \cos a - y \sin a,$$
$$y \text{ becomes } x \sin a + y \cos a.$$

3. For change of rectangular to oblique co-ordinates,

$$x \text{ becomes } x \cos a + y \sin \beta,$$
$$y \text{ becomes } x \sin a + y \cos \beta.$$

4. For change of oblique to rectangular co-ordinates,

$$x \text{ becomes } \frac{x \sin (\omega - a)}{\sin \omega} - \frac{y \cos (\omega - a)}{\sin \omega},$$

$$y \text{ becomes } \frac{x \sin a}{\sin \omega} + \frac{y \cos a}{\sin \omega}.$$

5. For change of rectangular to polar co-ordinates,

$$x \text{ becomes } r \cos \theta,$$
$$y \text{ becomes } r \sin \theta.$$

6. Distance between two points $(x_1\,y_1)$ and $(x_2\,y_2)$

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

EQUATIONS TO THE STRAIGHT LINE.

Where a and b are its intercepts on the axes of x and y.
p its perpendicular distance from the origin.
α the inclination of p to the axis of x, or to the initial line, and β its inclination to the axis of y.
m the tangent of the angle which it makes with the positive part of the axis of x.
n the ratio of the sines of the angles which it makes with the axes of x and y.

$$\left.\begin{array}{ll} 7. & y = mx + b \\ 8. & \dfrac{x}{a} + \dfrac{y}{b} = 1 \\ 9. & x \cos \alpha + y \sin \alpha = p \end{array}\right\} \text{Rectangular co-ordinates.}$$

$$\left.\begin{array}{ll} 10. & y = nx + b \\ 11. & \dfrac{x}{a} + \dfrac{y}{b} = 1 \\ 12. & x \cos \alpha + y \cos \beta = p \end{array}\right\} \text{Oblique co-ordinates.}$$

13. $\rho = p \sec (\theta - \alpha)$ Polar co-ordinates.

FORMS OF CERTAIN LINES REFERRED TO RECTANGULAR AXES.

14. Line through origin,

$$y = mx.$$

15. Line through $(x_1\,y_1)$,

$$y - y_1 = m\,(x - x_1).$$

16. Line through $(x_1\,y_1)$ and $(x_2\,y_2)$,

$$y - y_1 = \frac{y_1 - y_2}{x_1 - x_2}\,(x - x_1).$$

17. Line through $(x_1\ y_1)$ and making an angle $\tan^{-1} t$ with the line $y = m\,x + b$,

$$y - y_1 = \frac{m - t}{1 + m\,t}(x - x_1).$$

18. For intersection of lines $\left.\begin{aligned} y &= m\,x + b, \\ \text{and } y &= m'x + b', \end{aligned}\right\}$

$$x = \frac{b - b'}{m' - m} \text{ and } y = \frac{m'b - m\,b'}{m' - m}.$$

19. Angle between lines $\left.\begin{aligned} y &= m\,x + b, \\ \text{and } y &= m'x + b', \end{aligned}\right\}$

$$= \tan^{-1}\left(\frac{m - m'}{1 + m\,m'}\right).$$

20. Condition of parallelism, $m = m'$.

21. Condition of perpendicularity, $m = -\dfrac{1}{m'}$.

22. Perpendicular from $(x_1\ y_1)$ on the line $y = m\,x + b$

$$= \frac{y_1 - m\,x_1 - b}{\sqrt{1 + m^2}}.$$

THE CIRCLE.

(r = radius).

23. Equation to the circle, centre being origin,

$$x^2 + y^2 = r^2.$$

24. General equation, h and k being co-ordinates of centre,

$$(x - h)^2 + (y - k)^2 = r^2.$$

25. Equation to tangent at point $(x'\ y')$,

$$x\,x' + y\,y' = r^2.$$

26. Equation to normal,

$$y = \frac{y'}{x'} \cdot x.$$

THE PARABOLA.

Focus S. Vertex A. $AS = a$.

27. Equation to parabola, vertex being origin,

$$y^2 = 4ax.$$

28. Equation to parabola, origin being point (k, h) in the curve.

$$y = \frac{h}{2a}x - \frac{x^2}{4a}.$$

29. Polar equation to parabola, focus being pole,

$$\rho = \frac{2a}{1 - \cos\theta}.$$

30. Equation to parabola referred to any diameter and tangent at its extremity (P). PV, QV being co-ordinates of any point Q,

$$QV^2 = 4SP \,.\, PV.$$

31. Equation to tangent at point $(x'y')$,

$$yy' = 2a\,(x + x').$$

32. Equation to normal through $(x'y')$,

$$y - y' = -\frac{y'}{2a}(x - x').$$

General Properties of the Parabola.

33. Any point P is equidistant from focus and directrix.

34. Latus rectum $= 4a$.

35. Focal distance $SP = a + x$.

36. Subtangent $= 2x$.

37. Subnormal $= 2a$.

38. $SY^2 = SP \,.\, SA$ where $SY \perp$ tangent at P.

39. A diameter and the focal distance of its extremity make equal angles with the tangent at the same point.

40. Tangents at the extremity of any focal chord intersect at right angles in the directrix.

THE ELLIPSE.

Centre C. Foci S and H. Axis major $ACA' = 2a$. Axis minor $BCB' = 2b$. X the intersection of the axis major with the directrix. Eccentricity $= e$.

41. Equation to the ellipse, centre being origin,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

42. Equation to the ellipse, vertex A being origin,

$$y = \pm \frac{b}{a} \sqrt{2ax - x^2}.$$

43. Polar equation to the ellipse, centre being pole,

$$\rho = \frac{b}{\sqrt{1 - e^2 \cos^2 \theta}}.$$

44. Polar equation to the ellipse, focus S being pole,

$$\rho = \frac{a(1 - e^2)}{1 - e \cos \theta}.$$

45. Equation to the ellipse referred to conjugate diameters CP, CD; CV, QV being co-ordinates of Q,

$$QV^2 = \frac{CD^2}{CP^2} . PV . VG.$$

46. Equation to tangent at point $(x'y')$,

$$\frac{xx'}{a^2} + \frac{yy'}{b^2} = 1.$$

47. Equation to normal through $(x'y')$,

$$y - y' = \frac{a^2 y'}{b^2 x'} (x - x').$$

General Properties of the Ellipse.

$$\frac{\text{Focal distance of point } P}{\text{Distance of } P \text{ from the directrix}} = \text{constant} = e . (e < 1).$$

48.
$$e = \frac{\sqrt{a^2 - b^2}}{a}.$$

49. $$CX = \frac{a}{e}; \quad SX = \frac{a(1-e^2)}{e}.$$

50. $$CS = ae; \quad AS = a(1-e).$$

51. $$\text{Latus rectum} = \frac{2b^2}{a} = 2a(1-e^2).$$

52. $$SP = a + ex.$$

53. $$HP = a - ex.$$

54. $$SP + HP = \text{constant} = 2a.$$

55. $$\text{Subtangent} = \frac{a^2 - x'^2}{x'}.$$

56. $$\text{Subnormal} = \frac{b^2}{a^2} \cdot x'.$$

57. The ellipse becomes a parabola when $CS = \infty$; AS remaining finite.

58. The normal at any point bisects the angle between the focal distances of that point.

59. $$CP^2 + CD^2 = a^2 + b^2.$$

60. $$CD^2 = SP \cdot HP.$$

61. $$SY^2 = b^2 \cdot \frac{SP}{HP} \text{ where } SY \perp \text{ tangent at } P.$$

62. The tangents at the extremities of any focal chord intersect in the directrix.

63. The tangent at the extremity of any diameter is parallel to the corresponding conjugate diameter.

THE HYPERBOLA.

Centre C. Foci S and H. Transverse axis $ACA' = 2a$. Conjugate axis $BCB' = 2b$. Eccentricity $= e$.

64. Equation to the hyperbola, centre being origin,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

65. Equation to the hyperbola, vertex A being origin,

$$y = \pm \frac{b}{a} \sqrt{2ax + x^2}.$$

66. Polar equation to the hyperbola, centre being pole,

$$\rho = \frac{b}{\sqrt{e^2 \cos^2 \theta - 1}}.$$

67. Polar equation to the hyperbola, focus being pole,

$$\rho = \frac{a(e^2 - 1)}{1 - e \cos \theta}.$$

68. Equation to the hyperbola referred to conjugate diameters CP, CD; CV, QV being co-ordinates of Q,

$$QV^2 = \frac{CD^2}{CP^2} \cdot PV \cdot VG.$$

69. Equation to the hyperbola referred to its asymptotes,

$$xb = \frac{1}{4}(a^2 + b^2).$$

70. Equation to the tangent at point $(x' y')$,

$$\frac{xx'}{a^2} - \frac{yy'}{b^2} = 1.$$

71. Equation to the normal through $(x' y')$,

$$y - y' = -\frac{a^2}{b^2}\frac{y'}{x'}(x - x').$$

General Properties of the Hyperbola.

$$\frac{\text{Focal distance of point } P}{\text{Distance of } P \text{ from directrix}} = \text{constant} = e, \ (e > 1).$$

72. $$e = \frac{\sqrt{a^2 + b^2}}{a}.$$

73. $$CX = \frac{a}{e}; \quad SX = \frac{a(e^2 - 1)}{e}.$$

74. $$CS = ae; \quad AS = a(e - 1).$$

75. $$\text{Latus rectum} = \frac{2b^2}{a} = 2a(e^2 - 1).$$

76. $$SP = ex - a.$$

77. $$HP = ex + a.$$

78. $$HP - SP = \text{constant} = 2a.$$

79. $$\text{Subtangent} = \frac{x'^2 - a^2}{x'}.$$

80. $$\text{Subnormal} = \frac{b^2}{a^2} \cdot x'.$$

81. The hyperbola becomes a parabola when $CS = \infty$; AS remaining finite.

82. The normal at any point bisects the exterior angle between the focal distances of that point.

83. $$CP^2 - CD^2 = a^2 - b^2.$$

84. $$CD^2 = SP \cdot HP.$$

85. $$SY^2 = b^2 \cdot \frac{SP}{HP} \text{ where } SY \perp \text{ tangent at } P.$$

General Equation to the Section of a Cone

Made by a plane inclined to the axis of the cone at angle $\pi - \theta - \alpha$, at a perpendicular distance $d \sin \theta$ from the vertex: 2α being the vertical angle of the cone.

86. $$y^2 = \frac{2d \sin \alpha \sin \theta}{\cos \alpha} x - \frac{\sin \theta \cdot \sin (2\alpha + \theta)}{\cos^2 \alpha} \cdot x^2.$$

EQUATIONS TO CERTAIN LOCI.

The Cycloid.

87. $$\left.\begin{aligned} x &= r(\theta - \sin \theta), \\ y &= r(1 - \cos \theta); \end{aligned}\right\}$$

or, $$x = r \operatorname{vers}^{-1} \frac{y}{a} - \sqrt{2ry - y^2}.$$

The Epicycloid.

88. $$\left.\begin{aligned} x &= (a+r)\cos\theta - r\cos\left(\frac{a+r}{r}\,.\,\theta\right) \quad (1).\\ y &= (a+r)\sin\theta - r\sin\left(\frac{a+r}{r}\,.\,\theta\right) \quad (2).\end{aligned}\right\}$$

The Hypocycloid.

89. $$\left.\begin{aligned} x &= (a-r)\cos\theta + r\cos\left(\frac{a-r}{r}\,.\,\theta\right) \quad (1).\\ y &= (a-r)\sin\theta - r\sin\left(\frac{a-r}{r}\,.\,\theta\right) \quad (2).\end{aligned}\right\}$$

The Companion to the Cycloid.

90. $$\left.\begin{aligned} x &= r(1-\cos\theta) \quad (1).\\ y &= r\theta \quad (2).\end{aligned}\right\}$$

The Trochoid.

91. $$\left.\begin{aligned} x &= r(\theta - n\sin\theta) \quad (1).\\ y &= r(1 - n\cos\theta) \quad (2).\end{aligned}\right\}$$

The Epitrochoid.

92. $$\left.\begin{aligned} x &= (a+r)\cos\theta - d\cos\left(\frac{a+r}{r}\,.\,\theta\right) \quad (1).\\ y &= (a+r)\sin\theta - d\sin\left(\frac{a+r}{r}\,.\,\theta\right) \quad (2).\end{aligned}\right\}$$

The Hypotrochoid.

93. $$\left.\begin{aligned} x &= (a - r)\cos\theta + d\cos\left(\frac{a-r}{r}\,.\,\theta\right) \quad (1). \\ y &= (a - r)\sin\theta - d\sin\left(\frac{a-r}{r}\,.\,\theta\right) \quad (2). \end{aligned}\right\}$$

The Cardioid.

94. $$\rho = 2r\,(1 - \cos\theta).$$

The Spiral of Archimedes.

95. $$\rho = \frac{a}{2\pi}\,\theta.$$

The Logarithmic Spiral.

96. $$\rho = a^{\theta}.$$

The Catenary.

97. $$\left.\begin{aligned} y + a &= \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right) \\ s &= \frac{a}{2}\left(e^{\frac{x}{a}} - e^{-\frac{x}{a}}\right) \end{aligned}\right\}$$

The lowest point of the curve being origin; the axis of y vertical; a the length of chain whose weight = tension at the lowest point.

The Cissoid of Diocles.

98. $$y^2\,(2r - x) = x^3.$$

The Conchoid of Nicomedes.

99. $$x^2y^2 = (a^2 - y^2)\,(b + y)^2.$$

The Witch of Agnesi.

100. $$xy^2 = 4r^2\,(2r - x).$$

The Lemniscate of Bernoulli.

101. $$\{y^2 + (a + x)^2\}\,\{y^2 + (a - x)^2\} = c^4.$$

The Logarithmic Curve.

102. $$y = a^x.$$

The Quadratrix of Dinostratus.

103. $$y = (r - x)\tan\left(\frac{\pi}{2}\,.\,\frac{x}{r}\right).$$

MENSURATION.

r denotes the radius of a circle.

$$\pi = \frac{\text{circumference of a circle}}{\text{diameter}}.$$

Approximate values of π,

(1) 3·14159265.
(2) 3·14159.
(3) $3\frac{1}{7}$.

1. If A = interior angle of a regular figure of n sides,

$$A^\circ = \frac{n-2}{n}\,.\,180^\circ.$$

In a pentagon $A = 108^\circ$.
In a hexagon $A = 120^\circ$.
In an octagon $A = 135^\circ$.

2. Chord of a circle subtending A°,

$$= 2r\sin\frac{A}{2}.$$

ARCS AND PERIMETERS.

3. Arc of a circle subtending A°,

$$= \frac{A}{180} \cdot \pi r.$$

4. Perimeter of a circle,

$$= 2 \pi r.$$

5. Arc of a parabola, measured from vertex,

$$= \sqrt{a x + x^2} + a \log_e \frac{\sqrt{x} + \sqrt{a + x}}{\sqrt{a}}.$$

6. Perimeter of ellipse, semi axes a and b,

$$= \pi (a + b) \text{ approximately.}$$

7. Arc of cycloid measured from vertex,

$$= \sqrt{8 r x}.$$

8. Length of cycloid,

$$= 8 r.$$

9. Arc of catenary measured from lowest point,

$$= \frac{a}{2}\left(e^{\frac{x}{a}} - e^{-\frac{x}{a}}\right)$$

$$= \sqrt{y^2 - a^2}$$

10. Length of the catenary

$$= a\left(e^{\frac{l}{a}} - e^{-\frac{l}{a}}\right)$$

Where $2 l$ = horizontal distance between the points of attachment.

AREAS.

11. Area of rectangle, sides a, b,

$$= a \cdot b.$$

12. Area of parallelogram, sides a, b including $\angle A$,

$$= a b \sin A.$$

13. Area of triangle, altitude p, base c,

$$= \frac{1}{2} c \cdot p,$$

$$= \frac{1}{2} ab \sin C,$$

$$= \sqrt{s (s-a)(s-b)(s-c)}.$$

14. Area of trapezium, altitude p, parallel sides a, b,

$$= \frac{1}{2} p (a+b).$$

15. Area of polygon of n sides (a),

$$= n \cdot \frac{a^2}{4} \cdot \cot \frac{180}{n}.$$

16. Area of circle,

$$= \pi r^2.$$

17. Area of annulus, exterior and interior radii, R, r,

$$= \pi (R^2 - r^2).$$

18. Area of circular sector, $\angle A$,

$$= \frac{1}{2} \text{ arc} \times \text{radius},$$

$$= \pi r^2 \cdot \frac{A}{360}.$$

19. Area of circular segment,

= area of sector — area of triangle,

$$= \pi r^2 \cdot \frac{A}{360} - \frac{1}{2} r^2 \sin A.$$

20. Area of cycloid,

= 3 × area of the generating circle,

$$= 3 \pi r^2.$$

21. Area of parabola cut off by a double ordinate $2y$,

$= \frac{2}{3}$ of parallelogram with same base and height,

$$= \frac{4}{3} \cdot xy.$$

22. Area of ellipse semi axes a, b,

$$= \pi a b.$$

23. *Simpson's Rule for determining the approximate area of any curvilinear figure.*

Let the base be divided into any number of parts each $= D$, and from the points of section draw perpendiculars to meet the curve. Let

F denote the first, L the last perpendicular.
E = sum of all the even perpendiculars.
O = sum of all the odd perpendiculars.

Then area $= \frac{1}{3} D (F + L + 4 E + 2 O)$.

SURFACES.

24. Surface of a sphere,

$$= 4 \pi r^2.$$

25. Surface of cylinder, height h,

$$= 2 \pi r h.$$

26. Surface of prism, perimeter of base P,

$$= P . h.$$

27. Surface of cone or pyramid,

$$= \frac{1}{2} \text{ perimeter of base} \times \text{slant height.}$$

28. Surface of spherical segment, height h,

$$= 2 \pi r h.$$

29. Surface of a paraboloid,

$$= \frac{8 \pi}{3} \sqrt{a} \left(\sqrt{(x + a)^3} - \sqrt{a^3} \right).$$

30. Surface of prolate spheroid,

$$= \frac{2\pi ab}{e}\left\{\sin^{-1} e + e\sqrt{1-e^2}\right\}.$$

31. Surface of oblate spheroid,

$$= 2\pi a^2\left\{1 + \frac{1-e^2}{2e}\log\left(\frac{1+e}{1-e}\right)\right\}.$$

VOLUMES.

32. Vol. of rectangular parallelopiped, sides a, b, c,

$$= abc.$$

33. Vol. of sphere,

$$= \frac{2}{3} \times \text{circumscribing cylinder},$$

$$= \frac{4}{3}\pi r^3.$$

34. Vol. of spherical segment, height h,

$$= \frac{\pi}{3}(3r - h)h^2.$$

35. Vol. of cylinder or prism,

$$= \text{area of base} \times \text{height},$$

$$= A \,.\, h.$$

36. Vol. of cone or pyramid,

$$= \frac{1}{3} \times \text{circumscribing cylinder or prism},$$

$$= \frac{1}{3} \,.\, A \,.\, h.$$

37. Vol. of paraboloid,

$$= \frac{1}{2} \times \text{circumscribing cylinder},$$

$$= \frac{1}{2}\pi x y^2.$$

38. Vol of prolate spheroid,

$$= \frac{2}{3} \times \text{circumscribing cylinder,}$$

$$= \frac{4\pi a b^2}{3}.$$

39. Vol. of oblate spheroid,

$$= \frac{2}{3} \times \text{circumscribing cylinder,}$$

$$= \frac{4\pi a^2 b}{3}.$$

DIFFERENTIAL CALCULUS.

DEFINITIONS.

I. *Function of one or more variable quantities.*—An algebraical expression whose value depends upon that of one or more of the variable quantities which it involves.

II. *Limiting value of a function.*—That quantity from which the function may be made to differ as little as we please by making the difference between the variable and some particular value of it as small as we please.

General Notation.

a, b, c denote constant quantities *generally*.
x, y, z „ variable quantities *generally*.
u, v, w „ functions of one or more variables.
$f(x)$ „ function of x.

$f(x+h)$ denote the value of $f(x)$ when x becomes $x+h$.
dx, du, dv „ differentials of x, u, v.
$\frac{du}{dx}$, $\frac{dv}{dx}$, $\frac{dw}{dx}$ „ differential coefficients of u, v, w, with respect to x.

$$\frac{du}{dx} = \left\{\frac{f(x+h)-f(x)}{h}\right\}_{h=0}.$$

FORMULÆ FOR DIFFERENTIATION.

If $u=f(y)$ and $y=f(x)$,

$$\frac{du}{dx} = \frac{du}{dy}\cdot\frac{dy}{dx}.$$

1. $$\frac{dC}{dx} = 0 \text{ where } C = \text{constant}.$$

2. $$\frac{dx}{dx} = 1 \qquad \frac{d(Cx)}{dx} = C.$$

3. $$\frac{d(u+v+w)}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx}.$$

4. $$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}.$$

5. $$\frac{d(uvw)}{dx} = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}.$$

6. $$\frac{du^n}{dx} = nu^{n-1}\frac{du}{dx} \text{ where } n = \text{constant}.$$

7. $$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{d(uv^{-1})}{dx}.$$

$$= \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}.$$

8. $$\frac{da^u}{dx} = \log_e a \cdot a^u \frac{du}{dx}.$$

9. $$\frac{de^x}{dx} = e^x.$$

10. $$\frac{d(\log_a u)}{dx} = \frac{1}{\log_e a} \cdot \frac{1}{u} \cdot \frac{du}{dx}.$$

11. $$\frac{d(\log_e x)}{dx} = \frac{1}{x}.$$

12. $$\frac{du^v}{dx} = u^v \left(\log_e u \frac{dv}{dx} + \frac{v}{u} \frac{du}{dx}\right).$$

13. $$\frac{d \sin u}{dx} = \cos u \cdot \frac{du}{dx}.$$

14. $$\frac{d \cos u}{dx} = - \sin u \cdot \frac{du}{dx}.$$

15. $$\frac{d \tan u}{dx} = \sec^2 u \cdot \frac{du}{dx}.$$

16. $$\frac{d \cot u}{dx} = - \operatorname{cosec}^2 u \cdot \frac{du}{dx}.$$

17. $$\frac{d \sec u}{dx} = \sec u \cdot \tan u \cdot \frac{du}{dx}.$$

18. $$\frac{d \operatorname{cosec} u}{dx} = - \operatorname{cosec} u \cdot \cot u \cdot \frac{du}{dx}.$$

19. $$\frac{d \sin^{-1} u}{dx} = \frac{1}{\sqrt{1 - u^2}} \cdot \frac{du}{dx}.$$

20. $$\frac{d \cos^{-1} u}{dx} = - \frac{1}{\sqrt{1 - u^2}} \cdot \frac{du}{dx}.$$

21. $$\frac{d \tan^{-1} u}{dx} = \frac{1}{1 + u^2} \cdot \frac{du}{dx}.$$

22. $$\frac{d \cot^{-1} u}{dx} = -\frac{1}{1+u^2} \cdot \frac{du}{dx}.$$

23. $$\frac{d \sec^{-1} u}{dx} = \frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx}.$$

24. $$\frac{d \operatorname{cosec}^{-1} u}{dx} = -\frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx}.$$

25. $$\frac{d \operatorname{vers}^{-1} u}{dx} = \frac{1}{\sqrt{2u-u^2}} \cdot \frac{du}{dx}.$$

THEOREMS.

26. *Taylor's Theorem.* Where u denotes $f(x)$,

$$f(x+h) = u + \frac{du}{dx} \cdot \frac{h}{1} + \frac{d^2 u}{dx^2} \cdot \frac{h^2}{1 \cdot 2} + \frac{d^3 u}{dx^3} \cdot \frac{h^3}{1 \cdot 2 \cdot 3} + \\ + \ldots + \frac{d^n u}{dx^n} \frac{h^n}{\underline{|n}} + .$$

27. *Maclaurin's Theorem.* Where $f(0)$, $f'(0)$, $f''(0) \ldots$ denote the respective values of u, $\frac{du}{dx}$, $\frac{d^2 u}{dx^2}$, ... when $x = 0$,

$$f(x) = f(0) + f'(0) \cdot \frac{x}{1} + f''(0) \cdot \frac{x^2}{1 \cdot 2} + f'''(0) \frac{x^3}{1 \cdot 2 \cdot 3} + \ldots$$

28. *Liebnitz's Theorem.* Symbolical expression,

$$\left(\frac{d}{dx}\right)^n (uv) = \left(\frac{d}{dx} + \frac{d'}{dx}\right)^n uv.$$

29. If $u = f(y, z)$, y, z being functions of x,

$$\frac{du}{dx} = \left(\frac{du}{dy}\right) \cdot \frac{dy}{dx} + \left(\frac{du}{dz}\right) \cdot \frac{dz}{dx}.$$

30. If $u = f(x, y) = 0$, y being a function of x,

$$\left(\frac{du}{dx}\right) + \left(\frac{du}{dy}\right) \cdot \frac{dy}{dx} = 0.$$

31. If $u = f(x, y, z) = 0$, x and y being independent variables,

$$\left.\begin{aligned}\left(\frac{du}{dx}\right) + \left(\frac{du}{dz}\right) \cdot \frac{dz}{dx} = 0 \\ \left(\frac{du}{dy}\right) + \left(\frac{du}{dz}\right) \cdot \frac{dz}{dy} = 0\end{aligned}\right\} .$$

RULES OF MAXIMA AND MINIMA.

32. $u = f(x)$ *may* be made a maximum or a minimum by the value $x = a$ derived from the equation,

$$\frac{du}{dx} = 0.$$

33. $x = a$ renders u a *maximum* or a *minimum* according as

$$\left\{\frac{d^2u}{dx^2}\right\}_{x = a} \text{ is } \textit{negative} \text{ or } \textit{positive}.$$

34. If $\left\{\frac{d^2 u}{d x^2}\right\}_{x = a} = 0$, $\left\{\frac{d^3 u}{d x^3}\right\}_{x = a} = 0$ is a condition, and $f(a)$ is a *maximum* or a *minimum* according as $\left\{\frac{d^4 u}{dx^4}\right\}_{x = a}$ is *negative* or *positive*. So on generally.

35. Maxima and minima values of implicit functions,

$$u = f(x, y) = 0,$$

y *may* be made a maximum or a minimum by the value $x = a$ derived from the simultaneous equations,

$$\left.\begin{aligned}u &= 0 \\ \left(\frac{du}{dx}\right) &= 0\end{aligned}\right\} .$$

36. $x = a$ renders y a maximum or a minimum according as

$$\left\{\frac{\left(\frac{d^2u}{dx^2}\right)}{\left(\frac{du}{dy}\right)}\right\}_{x = a} \text{ is } \textit{positive} \text{ or } \textit{negative}.$$

37. Differential coefficents of Arc (s). Area (A); surface S); volume (V).

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$

$$\frac{dA}{dx} = y.$$

$$\frac{dS}{dx} = 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$

$$\frac{dV}{dx} = \pi y^2.$$

38. Equation to the tangent at point (x, y),

$$Y - y = \frac{dy}{dx}(X - x).$$

39. Equation to the normal at point (x, y),

$$(Y - y)\frac{dy}{dx} + X - x = 0.$$

40. Equation to the asymtote,

Where a and b denote the limits of $\frac{dy}{dx}$, and $x - y \cdot \frac{dx}{dy}$, when x increases indefinitely,

$$Y = a(X - b).$$

41. Radius of curvature (R),

$$R = -\frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}.$$

42. Spirals,

$$\frac{1}{p^2} = u^2 + \left(\frac{du}{d\theta}\right)^2.$$

43. Singular points of a curve whose equation is,

$$u = f(xy) = 0.$$

CONDITIONS OF—

(1) *Multiple points,*

$$\frac{dy}{dx} = \frac{0}{0} \quad \begin{cases} \left(\dfrac{du}{dx}\right) = 0 & (1). \\ \left(\dfrac{du}{dy}\right) = 0 & (2). \end{cases}$$

(2) *Points of inflexion,*

$\dfrac{d^2y}{dx^2}$ changes sign for a small increase and decrease of x.

(3) *Conjugate points,*

$\dfrac{dy}{dx} = \dfrac{0}{0}$, having *impossible* values.

INTEGRAL CALCULUS.

DEFINITIONS AND GENERAL NOTATION.

Indefinite Integral. $\int u\,dx$—That function of x which becomes u when differentiated with respect to x.

$$\int u\,dx = f(x) + C. \quad C \text{ being constant.}$$

Definite Integral. $\int_a^b u\,.\,dx$—The value of the indefinite integral $\int u\,dx$ between the limits $x = a$, and $x = b$.

$$\int_a^b u\,dx = f(b) - f(a).$$

Formulæ for Immediate Integration.

1. $$\int C \cdot \frac{du}{dx} \cdot dx = C \cdot u.$$

2. $$\int (\pm u \pm v \pm w)\, dx = \pm \int u dx \pm \int v dx \pm \int w dx.$$

3. $$\left.\begin{aligned} \int \left(C \cdot u^n \frac{du}{dx}\right) dx &= C \cdot \frac{u^{n+1}}{n+1} \\ \int x^n\, dx &= \frac{x^{n+1}}{n+1} \end{aligned}\right\} \text{Except } n = 1.$$

4. $$\int \left(\frac{1}{u} \cdot \frac{du}{dx}\right) dx = \log_e u.$$

5. $$\int \frac{dx}{x} = \log x.$$

6. $$\int a^x\, dx = \frac{a^x}{\log_e a}.$$

7. $$\int e^x\, dx = e^x.$$

8. $$\int \sin x\, dx = -\cos x.$$

9. $$\int \cos x\, dx = \sin x.$$

10. $$\int \sec^2 x\, dx = \tan x.$$

11. $$\int \operatorname{cosec}^2 x\, dx = -\cot x.$$

12. $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}.$$

13. $$\int \left(-\frac{dx}{\sqrt{a^2 - x^2}}\right) = \cos^{-1} \frac{x}{a}.$$

14. $$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}.$$

15. $$\int\left(-\frac{dx}{a^2+x^2}\right)=\frac{1}{a}\cot^{-1}\frac{x}{a}.$$

16. $$\int\frac{dx}{x\sqrt{x^2-a^2}}=\frac{1}{a}\sec^{-1}\frac{x}{a}.$$

17. $$\int\left(-\frac{dx}{x\sqrt{x^2-a^2}}\right)=\frac{1}{a}\operatorname{cosec}^{-1}\frac{x}{a}.$$

18. $$\int\frac{dx}{\sqrt{2ax-x^2}}=\operatorname{vers}^{-1}\frac{x}{a}.$$

19. $$\int\frac{dx}{\sqrt{x^2\pm a^2}}=\log\left(x+\sqrt{x^2\pm a^2}\right).$$

20. $$\int\frac{dx}{x\sqrt{a^2\pm x^2}}=-\frac{1}{a}\log\left(\frac{a+\sqrt{a^2\pm x^2}}{x}\right).$$

21. $$\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\log\frac{a+x}{a-x}.$$

22. $$\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\log\frac{x-a}{x+a}.$$

23. $$\int\sqrt{x^2\pm a^2}\,.\,dx=\frac{x}{2}\sqrt{x^2\pm a^2}\pm\frac{a^2}{2}\log\left(x+\sqrt{x^2\pm a^2}\right).$$

24. $$\int\sqrt{a^2-x^2}\,.\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}.$$

25. $$\int\sqrt{x^2+2ax}\,.\,dx=\frac{x+a}{2}\sqrt{x^2+2ax}$$
$$-\frac{a^2}{2}\log\left(x+a+\sqrt{x^2+2ax}\right).$$

26. $$\int\sqrt{2ax-x^2}\,.\,dx=\frac{x-a}{2}\sqrt{2ax-x^2}$$
$$+\frac{a^2}{2}\sin^{-1}\frac{x-a}{a}.$$

Formula for Integration by Substitution.

27. $$\int u\, dx = \int \left(u \frac{dx}{dy}\right) dy.$$

Formula for Integration by Parts.

28. $$\int \left(u \frac{dv}{dx}\right) dx = uv - \int \left(v \frac{du}{dx}\right) dx.$$

Formulæ of Reduction.

Where $a + bx^n = X$.

29. $$\int x^{m-1} X^p\, dx = \frac{X^p\, x^m}{m} - \frac{bnp}{m} \int x^{m+n-1} X^{p-1} dx.$$

30. $$\int x^{m-1} X^p\, dx = \frac{x^{m-n} X^{p+1}}{bn\,(p+1)} - \frac{m-n}{bn\,(p+1)} \times \int x^{m-n-1} X^{p+1}\, dx.$$

31. $$\int x^{m-1} X^p\, dx = \frac{x^m X}{am} - \frac{b\,(m+np+n)}{am} \times \int x^{m+n-1} X^p\, dx.$$

32. $$\int x^{m-1} X^p\, dx = \frac{x^{m-n} X^{p+1}}{b\,(m+np)} - \frac{(m-n)\,a}{b\,(m+np)} \times \int x^{m-n-1} X^p\, dx.$$

33. $$\int x^{m-1} X^p\, dx = \frac{x^m X^p}{m+np} + \frac{anp}{m+np} \times \int x^{m-1} X^{p-1}\, dx.$$

34. $$\int x^{m-1} X^p \, dx = -\frac{x^m X^{p+1}}{an(p+1)} + \frac{m+np+n}{an(p+1)} \times \int x^{m-1} X^{p+1} \, dx.$$

GEOMETRY OF THREE DIMENSIONS.

RECTANGULAR CO-ORDINATES.

DEFINITIONS.

I. *A Plane.*—(1) A surface in wl ich if any point be taken, it is equidistant from two given points. (2) A surface generated by a straight line moving always parallel to itself, and intersecting a given straight line.

II. *Co-ordinate Planes.*—Three fixed planes passing through the same fixed po nt called the *origin of Co-ordinates*, and intersecting one another two and two in straight lines called the *Axes of Co-ordinates.*

III. *Projections of a Point.*—The feet of the perpendiculars dropped from a proposed point upon the co-ordinate planes.

IV. *Traces of a Plane.*—The lines in which a plane intersect the co-ordinate planes.

V. *Projection of a Line upon a Plane.*—The locus of the *projections* of every point in the line.

VI. *Trace of a Line upon a Plane.*—The point in which the line meets the plane.

VII. *Projecting Plane.*—The plane which contains a proposed line and its projection.

General Notation.

(x, y, z) denotes the point whose co-ordinates are x, y, z.
$x\,y$ denotes the plane of the axes of x and y.
$x\,z$ denotes the plane of the axes of x and z.
$y\,z$ denotes the plane of the axes of y and z.

a, b, c, denote the portion of the co-ordinate axes intercepted between the origin and a proposed plane.

A, B denote the tangents of the angles at which the traces of a plane on xz, yz, are respectively inclined to the positive parts of the axes of x and y.

m, n denote the tangents of the angles at which the projections of a line on the planes xz, yz, are inclined to the axis of z.

1. Distance of a point (x, y, z) from the origin

$$= \sqrt{x^2 + y^2 + z^2}.$$

2. Distance between two points (x, y, z) and (x', y', z')

$$= \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}.$$

3. Equations to a plane,

(1) $$z = Ax + By + c,$$

(2) $$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

4. Equations to a straight line in space,

$$\left.\begin{aligned} x &= mz + a \\ y &= nz + b \end{aligned}\right\}.$$

5. Equations to a plane passing through point (x', y', z').

$$z - z' = A\,(x - x') + B\,(y - y').$$

6. Equations to a straight line through (x', y', z') and (x'', y'', z'').

$$\left.\begin{aligned} x - x' &= \frac{x' - x''}{z' - z''}\,(z - z') \\ y - y' &= \frac{y' - y''}{z' - z''}\,(z - z') \end{aligned}\right\}.$$

7. Condition of parallelism for a line and plane,

$Am + Bn - 1 = 0.$

8. Conditions of coincidence for a line and plane,

$$\left.\begin{aligned} Am + Bn - 1 &= 0 \\ Aa + Bb + c &= 0 \end{aligned}\right\}.$$

9. Conditions of perpendicularity for straight line and plane,

$$\left.\begin{aligned} A + m &= 0 \\ B + n &= 0 \end{aligned}\right\}.$$

10. Inclinations α, β, γ, to the co-ordinate axes of a line through the origin, parallel to a given line,

$$\cos\alpha = \frac{m}{\sqrt{1 + m^2 + n^2}}.$$

$$\cos\beta = \frac{n}{\sqrt{1 + m^2 + n^2}}.$$

$$\cos\gamma = \frac{1}{\sqrt{1 + m^2 + n^2}}.$$

11. Inclinations α, β, γ of a plane to the co-ordinate planes,

$$\cos\alpha = \frac{-A}{\sqrt{1 + A^2 + B^2}}.$$

$$\cos\beta = \frac{-B}{\sqrt{1 + A^2 + B^2}}.$$

$$\cos\gamma = \frac{1}{\sqrt{1 + A^2 + B^2}}.$$

12. Angle θ, between two lines,

$$\cos\theta = \frac{mm' + nn' + 1}{\sqrt{1 + m^2 + n^2}\,.\,\sqrt{1 + m'^2 + n'^2}};$$

or,

$$\cos\theta = \cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma'.$$

13. Angle θ, between two planes,

$$\cos\theta = \frac{1 + AA' + BB'}{\sqrt{1 + A^2 + B^2} \,.\, \sqrt{1 + A'^2 + B'^2}};$$

or,

$$\cos\theta = \cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma',$$

14. Condition of perpendicularity for two lines,

$$mm' + nn' + 1 = 0;$$

or,

$$\cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma' = 0.$$

15. Condition of perpendicularity for two planes,

$$1 + AA' + BB' = 0;$$

or,

$$\cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma' = 0.$$

16. Condition of two parallelisms for two lines or planes,

$$\cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma' = 1.$$

17. Angle θ, between a straight line and plane,

$$\sin\theta = \frac{1 - Am - Bn}{\sqrt{1 + m^2 + n^2} \,.\, \sqrt{1 + A^2 + B^2}}.$$

18. Distance of a point (x', y', z') from a plane

$$= \frac{z' - Ax' - By' - c}{\sqrt{1 + A^2 + B^2}}.$$

19. The shortest distance between two straight lines is the line perpendicular to both.

20. The length of the projection of a limited line upon a plane, is equal to the length of the line multiplied by the cosine of its inclination to the plane.

21 The length of the projection of a limited line upon any other line, is equal to the length of the line multiplied by the cosine of the angle between them.

22. The area of the projection of any plane surface upon a plane, is equal to the area of the surface, multiplied by the cosine of the angle between the planes.

23. The square of the area of any plane surface is equal to the sum of the squares of the areas of its projections on the co-ordinate planes.

SURFACES OF THE SECOND ORDER.

DEFINITIONS.

I. *Sphere.*—The surface is such that every point in it is equidistant from the centre.

II. *Cylinder.*—The surface is generated by an indefinite straight line carried round the perimeter of a given curve, and always remaining parallel to a given straight line.

III. *Cone.*—The surface is generated by an indefinite straight line carried round the perimeter of a given curve, and always passing through a fixed point.

IV. *Paraboloid.*—The surface is generated by the revolution of a parabola about its axis.

V. *Spheroid, prolate or oblate.*—The surface is generated by the revolution of an ellipse about its *major* or *minor* axis respectively.

VI. *Hyperboloid.*—The surface is generated by the revolution of a hyperbola about one of its axes.

VII. *Ellipoid.*—The surface is generated by a variable ellipse which always moves parallel to itself with its axes in two fixed planes at right angles to each other, and vertices in two *ellipses* in those planes, and having a common axis coincident with their intersection.

VIII. *Hyperboloid of one or two sheets.*—The surface is generated by a variable ellipse which always moves parallel to itself

with its axes in two fixed planes at right angles to each other, and having its vertices in two *hyperbolas* in those planes having a common *conjugate* or *transverse* axis *respectively*, coincident with their intersection.

IX. *Elliptic or hyperbolic paraboloid.*—The surface is generated by a parabola which moves with its axis in a fixed plane to which it always remains perpendicular, and parallel to the axis of another parabola along which its vertex moves; the concavities of the parabolas being turned towards the *same* or *opposite* parts respectively.

SURFACES.

24. Equation to the surface of a sphere, radius r, co-ordinates of centre h, k, l.

$$(x - h)^2 + (y - k)^2 + (z - c)^2 = r^2.$$

25. Equation to the surface of an oblique cylinder with circular base,

$$(y - nz)^2 = 2r(x - mz) - (x - mz)^2.$$

26. Equation to the surface of an oblique cone with circular base, h, k, l co-ordinates of the vertex,

$$(kz - ly)^2 = 2r(z - l)(hz - lx) - (hz - lx)^2.$$

27. Equation to the surface of a paraboloid, latus rectum $4a$,

$$y^2 + z^2 = 4ax.$$

28. Equation to the surface of a prolate or oblate spheroid semiaxis of the generating ellipse a, b,

$$\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1, \text{ or } \frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} = 1.$$

29. Equation to the surface of a hyperboloid,

$$\frac{x^2 + y^2}{a^2} - \frac{z^2}{b^2} = 1, \text{ or } \frac{x^2}{a^2} - \frac{y^2 + z^2}{b^2} = 1.$$

30. Equation to the surface of an ellipsoid,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

31. Equations to the surface of hyperboloid of *one* or *two* sheets ($\pm$)

$$\frac{x^2}{a^2} \pm \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1.$$

32. Equations to the surface of the elliptic or hyperbolic paraboloid ($\pm$),

$$x = \frac{y^2}{4a} \pm \frac{z^2}{4a'}.$$

33. Tangent plane at point (x, y, z) of a surface,

$$Z - z = \left(\frac{dz}{dx}\right)(X - x) + \left(\frac{dz}{dy}\right)(Y - y).$$

34. Normal at point (x, y, z),

$$\left.\begin{aligned} X - x + \left(\frac{dz}{dx}\right)(Z - z) &= 0 \\ Y - y + \left(\frac{dz}{dy}\right)(Z - z) &= 0 \end{aligned}\right\}.$$

35. Volume V of *any* solid,

$$V = \iint z\, dx\, dy.$$

36. Surface S of *any* solid,

$$S = \iint \sqrt{1 + \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2}\, dx\, dy.$$

NATURAL PHILOSOPHY.

STATICS.

Statical Definitions.

I. *Statics.*—That branch of mechanics which investigates the effects of force without reference to motion.

II. *Force.*—Any cause which changes, or tends to change, a body's state of rest or motion.

III. *Elements of force.*—Magnitude, direction, and point of application.

IV. *Resultant of two or more forces.*—That single force, which, acting by itself, produces the same effect as those forces acting together.

V. *Components of a force.*—Those forces which, acting together, produce the same effect as the single force.

VI. *Weight.*—That force which results from the earth's attraction upon a body.

VII. *Mass.*—The quantity of matter in a body, as indicated by its weight.

VIII. *Particle or material point.*—A portion of matter so small that it may be considered to be a point, without sensible error.

IX. *Inertia.*—That property which matter possesses of resisting change from rest to motion, or conversely.

X. *Rigid body.*—One that is, theoretically, incapable of being altered in its form or dimensions, by the action of any ordinary force.

XI. *Centre of parallel forces.*—That point where the resultant of a set of parallel forces always acts, whatever be the *directions* of the forces.

XII. *Centre of gravity of a body or system of bodies.*—That point

where the resultant of the weights of all the component particles *always* acts, whatever be the position of the body or system.

XIII. *Moment of a force about a point or line.*—The product of the number of units of force, and the number of units of length in the perpendicular distance of the line of direction of the force from the point or line.

XIV. *Elasticity.*—The tendency of matter to recover its original form and dimensions, after its component particles have been extended or compressed.

XV. *Tension.*—The force which is transmitted along a string, chain or rod, when one end is *pulled.*

XVI. *Thrust.*—The force which is transmitted through a rod, when one end is *pushed.*

XVII. *Cohesion or molecular attraction.*—The resistance offered by particles of matter to a force which tends to separate them.

XVIII. *Friction.*—The force which is brought into action by the *roughness* of two bodies in contact.

XIX. *Lever.*—A rigid rod, straight or bent, in which there is a fixed point or axis called the *Fulcrum,* about which it may freely turn in one plane.

XX. *Virtual velocity.*—The *space* through which the point of application of a force is moved, in the *direction* of the force, in consequence of a very small displacement.

Statical Axioms.

1. Any set of forces may be replaced by their *resultant.*

2. A single force may be replaced by its *components.*

3. The *resultant* of two or more forces acting on a particle in the *same direction,* is equal to their *sum.*

4. The *resultant* of two forces acting on a particle in *opposite directions,* is equal to their *difference,* and acts in the direction of the greater force.

5. The *direction* of the *resultant* of two equal forces acting on a particle, at an angle to each other, bisects that angle.

6. Two *equal* and *opposite* forces acting in the same straight line, on a particle, or on different points of a rigid body, balance each other.

7. If two forces balance each other, they are *equal* and *opposite.*

8. Any point of a free body may be supposed to become a fixed point if it has no tendency to move.

9. If two unconnected particles have no tendency to move towards or from each other, they may be supposed to become rigidly connected.

10. Forces which balance each other may be removed or applied at pleasure.

11. If several forces keep a body at rest, any one of them must be equal and opposite to the resultant of all the others.

12. Equal forces, acting in the same or parallel planes on different points of a rigid body, exert equal efforts to turn it about a fixed point or axis from which the perpendicular distances of their directions are equal.

Statical Principles.

Action and reaction are equal and opposite.

I. *Principle of the transmission of force.*—A force may be supposed to act at any point in its line of direction, if that point be in any way rigidly connected with the particle on which the force acts.

II. *Principle of the parallelogram of forces.*—If two adjacent sides of a parallelogram represent the magnitudes and directions of two forces acting at a point, then the intermediate diagonal will represent the magnitude and direction of the resultant of the forces.

III. *Theorem.*—The algebraic sum of the moments of two forces about any point in the plane in which they act, is equal to the moment of their resultant.

IV. *Principle of the triangle of forces.*—If three forces, acting at the same point, balance each other, they will be represented in magnitude and direction, by the sides, taken in order, of any triangle formed by lines drawn parallel to the directions of the forces: and conversely.

V. *Corollary to the principle of the triangle of forces.*—If three forces balance each other, they will be proportional to the sides of a triangle formed by lines perpendicular to the directions of the forces.

VI. *Principle of the concurrence of three balancing forces.*—The directions, produced if necessary, of three non-parallel forces which act upon a rigid body and balance each other, must be in the same plane, and pass through the same point.

VII. *Principle of parallel forces.*—The resultant of a set of parallel forces is equal to their algebraic sum; and its moment with respect to any point in the same plane with them, or with respect to any line in a plane perpendicular to their directions, is equal to the sum of the moments of the forces about the same point or line.

VIII. *Principle of the equality of moments.*—If a rigid body be kept at rest by any number of forces whose directions lie in the same or parallel planes, the sum of the moments of those forces which tend to turn the body in *one* direction about any point in that plane, or about any line perpendicular to those planes, is equal to the sum of the moments of those which tend to turn it in the *opposite* direction.

IX. *Principle of virtual velocities.*—If a system of bodies, in equilibrium under the action of any forces, be *very slightly* displaced, the sum of the products of the forces and their respective virtual velocities is equal to zero.

GENERAL PROPERTIES OF THE CENTRE OF GRAVITY.

1. A body, or system of bodies, has but one centre of gravity.

2. A body, or system of bodies rigidly connected will balance in *all* positions about its centre of gravity, if that point be supported.

3. The centre of gravity of a body must be in such a plane as divides it into two equal and similar parts.

4. The centre of gravity of a suspended body must lie in the vertical line through the point of suspension.

5. A body resting on any plane will *stand* or *fall*, according as the vertical line through its centre of gravity falls *within* or *without* the base of the body.

6. The centre of gravity, if it can move at all, always seeks its lowest possible position.

7. The equilibrium of a body is *stable* or *unstable*, according as its centre of gravity is in its *lowest* or *highest* position respectively; and is *neutral* when the centre of gravity neither ascends nor descends upon the body being disturbed.

8. *Guldinus' properties*—

(1) The surface generated by a curve line which revolves about a fixed axis, is equal to the product of the length of the line and the length of the path described by the centre of gravity of the curve.

(2) The volume generated by a plane area revolving about a fixed axis in its own plane, is equal to the product of the area and the length of the path described by the centre of gravity of the area.

THE MECHANICAL POWERS.

P denotes the applied or working power.

W „ resistance to be overcome.

$\frac{W}{P}$ „ mechanical advantage.

Levers.

First order $W >< P$.

Second „ $W > P$.

Third „ $W < P$.

Straight Lever.

1. Directions of P and W parallel.

P is to W inversely as the arms,

or,

$$\frac{W}{P} = \frac{a}{b}.$$

Any Lever.

2. P is to W inversely as the length of the perpendiculars from the fulcrum upon their directions,

or,

$$\frac{W}{P} = \frac{p}{q}$$

Common Balance.

3. Length $= 2l$. Weight $= W$. Centre of gravity G. Centre of suspension C.

Measure of Sensibility.

$$\frac{l^2}{CG \times W}.$$

False Balance.

4. True Weight $= \sqrt{\text{product of apparent weights.}}$

Wheel and Axle.

5. P is to W as the radius of the axle is to the radius of wheel,

or,

$$\frac{W}{P} = \frac{R}{r}.$$

Wheel and Differential Axle.

6. P is to W as the difference between the radii of the axles is to the diameter of the wheel,

or,

$$\frac{W}{P} = \frac{2R}{r - r'}.$$

Combination of Wheels and Axles.

7. P is to W as the product of the radii of all the axles is to the product of the radii of all the wheels,

or,

$$\frac{W}{P} = \frac{R_1 \,.\, R_2 \ldots R_n}{r_1 \,.\, r_2 \ldots r_n}.$$

Toothed Wheel.

8. $$\frac{W}{P} = \frac{R_1}{r_n} \times \frac{\text{product of the number of teeth in the wheels}}{\text{product of the number of teeth in the pinions}}.$$

Single Moveable Pulley, Strings parallel.

9. $$W = 2P.$$

Single Moveable Pulley, Strings including an angle A.

10. $$W = 2\cos\frac{A}{2} \, . \, P.$$

First system of Pulleys.

11. The same string passing round n moveable pulleys,

$$W = 2nP.$$

Second system of Pulleys,

12. Each of the n moveable pulleys hanging by a separate string,

$$W = 2^n \, . \, P.$$

Third system of Pulleys.

13. Each of the n strings being attached to the weight,

$$W = (2^n - 1) P.$$

Inclined Plane.

14. Inclination of the plane $= A$,
inclination of P to the plane $= \theta$,
reaction of the plane $= R$.

P (acting parallel to length) : W = height : length.
P (acting parallel to base) : W = height : base.

General conditions $\begin{cases} P\cos\theta = W\sin A. \\ W\cos A = R + P\sin\theta. \end{cases}$

The Screw.

15. P is to W as the longitudinal distance between the threads is to the circumference of the circle described by the point of application of the power,

or,

$$\frac{W}{P} = \frac{2\pi R}{d}.$$

STATICAL FORMULÆ.

Composition and Resolution of Forces.

16. $$R^2 = P^2 + Q^2 + 2PQ \cos A.$$

$$\sin\theta = \frac{Q}{R} . \sin A.$$

17. Rectangular resolution,

$$A = 90^\circ.$$
$$X = P\cos\theta.$$
$$Y = P\sin\theta.$$

$$\tan\theta = \frac{Y}{X}$$

$$R'^2 = (\Sigma X)^2 + (\Sigma Y)^2.$$

$$\tan\theta' = \frac{\Sigma Y}{\Sigma X}.$$

18. Triangle of forces,

$$P : Q : R = a : b : c.$$

Lami's Theorem.

19. $P : Q : R = \sin(Q, R) : \sin(P, R) : \sin(P, Q).$

20. *General Conditions of Equilibrium of Forces in the same Plane.*

$$(1) \quad \Sigma(X) = 0.$$
$$(2) \quad \Sigma(Y) = 0.$$
$$(3) \quad \Sigma(P \cdot p) = \Sigma(Q \cdot q).$$

21. *General Conditions of Equilibrium of Forces* NOT *in the same Plane.*

$$(1) \quad \Sigma(X) = 0.$$
$$(2) \quad \Sigma(Y) = 0.$$
$$(3) \quad \Sigma(Z) = 0.$$
$$(4) \quad \Sigma(Zy - Yz) = 0.$$
$$(5) \quad \Sigma(Xz - Zx) = 0.$$
$$(6) \quad \Sigma(Yx - Xy) = 0.$$

22. Parallel forces,

$$(1) \quad R = \Sigma(Y).$$
$$(2) \quad \bar{x} = \frac{\Sigma(Yx)}{\Sigma(Y)}.$$

23. Centres of gravity (G).

Arc of Circle.

$$CG = \frac{\text{rad.} \times \text{chd.}}{\text{arc}}$$

Arc of Semicircle.

24. $$CG = \frac{2r}{\pi}.$$

25. *Catenary.—See suspension bridges.*

Area of Triangle ACB . where $AO = BO$.

26. $$CG = \frac{2}{3} CO.$$

Area of Sector of Circle.

27. $$CG = \frac{2}{3} \cdot \frac{\text{rad.} \times \text{chd.}}{\text{arc}}.$$

Area oj Semicircle.

28. $$CG = \frac{4r}{3\pi}.$$

Area of Parabola.

29. $$AG = \frac{3}{5} \cdot x.$$

Area of Semi-ellipse.

30. $$CG = \frac{4}{3} \cdot \frac{b}{\pi} \text{ or } \frac{4}{3} \cdot \frac{a}{\pi}.$$

Surface of a Cone.

Vertex V. C. of G. of base C.

31. $$VG = \frac{2}{3} VC.$$

Surface of the Segment of a Sphere.

32. G bisects the height.

Volume of a Cone or Pyramid.

33. $$VG = \frac{3}{4} VC.$$

Volume of a Spherical Segment.

Height h. Centre of base C.

34. $$CG = \frac{3}{4} \cdot \frac{(2r - h)^2}{3r - h}.$$

Volume of a Hemisphere.

35. $$CG = \frac{3}{8} r.$$

Volume of a Paraboloid.

36. $$AG = \frac{2}{3} \cdot x.$$

Volume of a Semi-prolate Spheroid.

37. $$CG = \frac{3}{8} a.$$

Volume of a Semi-oblate Spheroid.

38. $$CG = \frac{3}{8} b.$$

39. General formulæ for the centre of gravity,

$$\bar{x} = \frac{\Sigma\,(mx)}{\Sigma\,(m)} \text{ or } \frac{\int x\,dm}{\int dm}.$$

$$\bar{y} = \frac{\Sigma\,(my)}{\Sigma\,(m)} \text{ or } \frac{\int y\,dm}{\int dm}.$$

$$\bar{z} = \frac{\Sigma (mz)}{\Sigma (m)} \text{ or } \frac{\int z\,dm}{\int dm}.$$

---

40. DEF. *Moment of Inertia* (I).—The product of the mass of a particle into the square of its distance from a fixed axis,

$$I = \Sigma (mr^2) \text{ or } \int r^2\,dm.$$

41. DEF. *Radius of Gyration* (K).—The distance from the axis of that point where, if the whole mass (M) be collected, the moment of inertia is unaltered,

$$MK^2 = I.$$

42. Straight line about a perpendicular axis through its extremity,

$$I = M \cdot \frac{l^2}{3}.$$

43. Circular area about a perpendicular line through its centre,

$$I = M\frac{r^2}{2}.$$

FRICTION.

44. *Laws of Friction.*

(1) The force of friction (F) is proportional to the mutual normal pressure (P) between the bodies in contact.

(2) The force of friction is independent of the extent of the surfaces in contact.

(3) The friction of motion is independent of the velocity.

45. DEF. *Coefficient of friction* (C).—The numerical quantity which expresses the ratio of the force of friction to the mutual normal pressure between the surfaces.

46. Def. *Angle of repose* (θ).—The greatest angle which a plane can make with the horizon, so that a given substance placed upon it, will be prevented from sliding by friction only.

$$F = C \cdot P$$
$$\tan \theta = C.$$

47. Def. *Angle of resistance* ($= \theta$).—The greatest angle which the direction of a force can make with the common normal to the surfaces at the point of contact without causing the one to slide over the other.

WORK.

DEFINITIONS.

I. *Work.*—The union of a continued pressure with a continued motion.

II. *Unit of Work.*—The labour required to raise a weight of one pound through a space of one foot.

III. *Horse Power* (*HP*).—The work expended in performing 33000 units of work per minute.

IV. *Modulus of a Machine.*—The numerical quantity expressing the ratio of the work performed to the work applied.

PRINCIPLES.

(1) "If a body be moved along any smooth surface, the work performed is the same as that due to the vertical height through which the body is raised."

(2) "The work expended in raising material of given weight and form, is expressed by the product of the weight of the material and the vertical distance through which its centre of gravity is raised."

(3) "*Equality of work.*—The resistance overcome by a perfect machine is equal to the work expended."

STRENGTH OF MATERIALS.

DEFINITIONS AND NOTATION.

I. *Proof Strength of Materials.* The weight or pressure which will produce the *greatest* strain without destroying the tenacity of the material.

II. *Absolute or ultimate Strength of Materials.*—The *breaking* weight or strain necessary to produce rupture.

III. *Load of Safety.*—A load or pressure less than that of proof strength, assumed in practice to provide for unforeseen contingencies.

IV. *Coefficients of "Proof Strength," "Absolute Strength," and "Safety."*—Constant quantities corresponding to the conditions of "Proof Strength," "Absolute Strength," and "Safety." They are determined experimentally, and depend upon the nature of the material.

V. *Tenacity and Resistance to Compression.*—The resistance of a material to rupture by extension or compression, respectively.

VI. *Modulus of Elasticity.*—The number of pounds required to double the length of a prism of one square inch sectional area, or to compress it to one half its length.

VII. *Neutral Surface.*—That layer of fibres in a solid body which is neither extended nor compressed under the action of a straining force, not in the direction of its axis.

VIII. *Neutral Axis.*—The intersection of the *neutral surface* with a given cross section of the body.

IX. *Neutral Line.*—The intersection of the *neutral surface* with a plane which divides the body, in the direction of its length, into two symmetrical portions.

Notation.

W = straining pressure in pounds.
A = sectional area of a prism or beam in square inches.
l = length of a beam or prism in inches.
B or b = breadth of a rectangular beam in inches.
D or d = depth of a rectangular beam in inches.
δl = elongation or compression of a prism.
$\triangle$ = greatest deflection of a beam.
C = coefficient of rupture or absolute strength referred to 1 lb. and 1 square inch.
E = modulus of elasticity.

Rupture by Extension or Compression.

48. $$W = C \, . \, A.$$

Rupture by Transverse Strain.

Beam fixed at one end and loaded at the other.

49. Solid rectangular beam,

$$W = C \, . \, \frac{bd^2}{6l}.$$

50. Solid cylindrical beam,

$$W = C \, . \, \frac{\pi r^3}{4l}.$$

51. Hollow rectangular beam,

$$W = C \, . \, \frac{BD^3 - bd^3}{6Dl}.$$

52. Hollow cylindrical beam,

$$W = C \, . \, \frac{\pi (R^4 - r^4)}{4Rl}.$$

Beam fixed at one end, and uniformly loaded.

53. Solid rectangular beam,

$$W = C \cdot \frac{b d^2}{3 l}.$$

54. Solid cylindrical beam,

$$W = C \cdot \frac{\pi r^3}{2 l}.$$

55. Hollow rectangular beam,

$$W = C \cdot \frac{B D^3 - b d^3}{3 D l}.$$

56. Hollow cylindrical beam,

$$W = C \cdot \frac{\pi (R^4 - r^4)}{2 R l}.$$

Beam supported at both ends and loaded in the middle.

57. Solid rectangular beam,

$$W = C \cdot \frac{2 b d^2}{3 l}.$$

58. Solid cylindrical beam,

$$W = C \cdot \frac{\pi r^3}{l}.$$

59. Hollow rectangular beam,

$$W = C \cdot \frac{2 (B D^3 - b d^3)}{3 D l}.$$

60. Hollow cylindrical beam,

$$W = C \cdot \frac{\pi (R^4 - r^4)}{R l}.$$

Beam supported at both ends and uniformly loaded.

61. Solid rectangular beam,

$$W = \frac{4bd^2}{3l}.$$

62. Solid cylindrical beam,

$$W = C \cdot \frac{2\pi r^3}{l}.$$

63. Hollow rectangular beam,

$$W = C \cdot \frac{4(BD^3 - bd^3)}{3Dl}.$$

64. Hollow cylindrical beam,

$$W = C \cdot \frac{2\pi(R^4 - r^4)}{Rl}.$$

Beam supported at both ends and loaded at a distance p from one end.

65. Solid rectangular beam,

$$W = C \cdot \frac{bd^2 l}{6p(l-p)}.$$

66. Solid cylindrical beam,

$$W = C \cdot \frac{\pi r^3 l}{4p(l-p)}.$$

67. Hollow rectangular beam,

$$W = C \cdot \frac{(BD^3 - bd^3)l}{6Dp(l-p)}.$$

68. Hollow cylindrical beam,

$$W = C \cdot \frac{\pi(R^4 - r^4)l}{4Rp(l-p)}.$$

Rupture of Cylindrical Columns by bending.

Hodgkinson's empirical formulæ.

Pressure in direction of length. Ends flat.

R, r expressed in inches.
l expressed in feet.
W expressed in tons.

69. Cast-iron, solid,

$$W = 44{\cdot}1 \, \frac{(2\,r)^{3{\cdot}6}}{l^{1{\cdot}7}}.$$

70. Cast-iron, hollow,

$$W = 44{\cdot}3 \, \frac{(2\,R)^{3{\cdot}6} - (2\,r)^{3{\cdot}6}}{l^{1{\cdot}7}}.$$

71. Wrought-iron, solid,

$$W = 133{\cdot}6 \, \frac{(2\,r)^{3{\cdot}6}}{l^2}.$$

72. Dry red deal, solid square,

$$W = 7{\cdot}81 \, \frac{(2\,r)^4}{l^2}.$$

Hooke's Law.

73. "The extension of elastic strings or rods is proportional to the tension,"

$$\delta\, l = \frac{W\,l}{E\,A}.$$

Deflections.

74. Beam *supported* at both ends and loaded in the middle,

$$\Delta = \frac{W\,l^3}{48\,E.I}.$$

75. Beam *supported* at both ends and loaded uniformly,

$$\Delta = \frac{5 W l^3}{384 E.I}.$$

76. Uniform beam, weight w, *fixed* at one end, and strained at the other,

$$\Delta = \frac{l^3}{3 E.I}(W + \frac{3}{8} w).$$

77. Beam *fixed* at both ends and loaded in the middle,

$$\Delta = \frac{W l^3}{192 E.I}.$$

78. Beam *fixed* at both ends and loaded uniformly,

$$\Delta = \frac{W l^3}{384 E.I}.$$

Values of I in the foregoing expressions for Δ.

79. Solid rectangular beam,

$$I = \frac{b d^3}{12}.$$

80. Solid cylindrical beam,

$$I = \frac{\pi r^4}{4}.$$

81. Hollow rectangular beam,

$$I = \frac{B D^3 - b d^3}{12}$$

82. Hollow cylindrical beam,

$$I = \frac{\pi (R^4 - r^4)}{4}.$$

RESISTANCE OF CYLINDERS AND SPHERES.

p = internal pressure in lbs. per square inch.
C = coefficient of resistance.
t = thickness of the metal.
r = radius of the cylinder or sphere.

Interior Surface of a Cylinder.

83. $$p = C\frac{t}{r}.$$

Base of a Cylinder, or Interior Surface of a Sphere.

84. $$p = C\left(2 + \frac{t}{r}\right)\frac{t}{r}.$$

ROOFS.

EXPLANATION OF TERMS.

I. *Trussed Roof.*—A rigidly connected framework of wooden or iron beams, *trussed* or *tied* together.

II. *Principal Rafters.*—Inclined beams in vertical planes, *trussed* together; meeting at the top of the roof, and supporting the covering.

III. *Common Rafters.*—Intermediate rafters resting on horizontal beams called *purlins.*

IV. *Tie Beam.*—The beam which connects each pair of principal rafters at their base.

V. *Stays.*—Cross beams connecting the principal rafters at intermediate points to prevent them from bending.

VI. *King Post.*—A vertical beam connecting the tie beam with the vertex of the rafters.

VII. *Queen Posts.*—Vertical beams or rods connecting the tie beam with the principal rafters at intermediate points.

VIII. *Struts.*—Inclined beams connecting the foot of the king post with the principal rafters at intermediate points.

DISTRIBUTION OF LOAD.

1st method. $\frac{W}{4}$ at B; $\frac{W}{2}$ at C and V.

2d method. $\frac{W}{3}$ at B and C; $\frac{2}{3}W$ at V,

where $W =$ weight of the roof supported by the rafter B, C, V.

THE ARCH.

EXPLANATION OF TERMS.

I. *Voussoirs.*—Component parts of the arch, in the form of truncated wedges, supported by their mutual pressures.

II. *Joints.*—The surfaces between the voussoirs.

III. *Key Stone.*—The highest voussoir in the arch.

IV. *Crown.*—The summit of the keystone.

V. *Piers or Abutments.*—Fixed surfaces supporting the lowest voussoirs in the arch.

VI. *Reins or Haunches.*—The portions of the arch on each side of the crown.

VII. *Springing Lines.*—The lines or beds below the reins at which the arch begins.

VIII. *Span.*—The breadth of the arch between the springing lines.

IX. *Surcharge.*—The superincumbent load of earth or masonry.

X. *Intrados.*—The *interior* outline of the arch.

XI. *Extrados.*—The *exterior* outline of the arch.

XII. *Line of Resistance.*—The line which joins all the resisting points of the resultant pressures upon the contiguous surfaces of the voussoirs.

XIII. *Line of Pressure.*—The line which joins all the consecutive intersections of the resultant pressures upon the joints of the voussoirs.

Conditions of Stability of the Arch.

85. (1) That no two portions of the arch may revolve on the edges of their common surface of contact; the *line of resistance* must lie between the *intrados* and *extrados.*

(2) That no two surfaces of contact may slide on each other, the angle between the *line of pressure* and the common normal to the surfaces must be less than the angle of repose.

86. *Rule for finding the Centre of Gravity of a mass of the arch intercepted by vertical planes through the point of rupture and highest point.*—Let $y_1, y_2 \ldots y_n$ be the vertical distances between n consecutive points in the intrados *or* extrados, the abscissa of which measured on a horizontal line from the point of rupture have a common difference $= d$.

Then

$$\bar{x} = \frac{d\{y_1 + (3n - 1)y_n + 6(y_2 + 2y_3 + 3y_4 + \ldots\}}{3\{y_1 + y_n + 2(y_2 + y_3 + y_4 + \ldots\}}.$$

87. *Rule for finding the point of rupture.*—Let H be the highest point of the extrados; R the assumed point of rupture; G the centre of gravity of the mass between R and H. Let the horizontal line HAB cut the vertical line AGC in A. Take AC to represent the units of weight in the mass RH. Complete the rectangle $BACD$. Then R will be the point of rupture if the diagonal AD produced *touches* the intrados at R.

SUSPENSION BRIDGES.

DEF. *Catenary*.—The curve assumed by a chain suspended by its extremities from two fixed points.

Lowest point of the curve, the origin.

Tangent at that point, the axis of x.

$2l$ = horizontal distance between the points of attachment.

$2S$ = whole length of the chain.

a = length of a portion of the chain whose weight represents the tension at the lowest point.

s = length of a portion of the chain between the origin and point (xy).

$2w$ = weight of the chain.

88. Equations to the catenary,

$$(1)\quad y = \frac{a}{2}\left(e^{\frac{x}{2a}} - e^{-\frac{x}{2a}}\right)^2.$$

$$(2)\quad s = \frac{a}{2}\left(e^{\frac{x}{a}} - e^{-\frac{x}{a}}\right).$$

Tension (T) *at any point.*

89. $$T = \frac{\sqrt{a^2 + s^2}}{S} \cdot w.$$

Inclination (θ) *of the catenary to the vertical at any point.*

90. $$\cot\theta = \frac{1}{2}\left(e^{\frac{x}{a}} - e^{-\frac{x}{a}}\right).$$

Relation between the length of the Catenary and the depth (D) of its lowest point below the points of attachment.

91. $$S = \sqrt{D\,(D + 2\,a.)}$$

Centre of Gravity.

92. $$\bar{y} = \frac{1}{2}\left\{D - a\left(1 - \frac{l}{S}\right)\right\}$$

DYNAMICS.

DEFINITIONS.

I. *Dynamics.*—That branch of mechanics which investigates the different kinds of motion produced by forces.

II. *Velocity or Rate of Motion.*—The number of units of space described by a body in a unit of time.

III. *Rate of Acceleration.*—The velocity generated in a unit of time by the *continued* action of a force.

IV. *Units of Time and Space.*—1 second and 1 foot.

V. *Mass.*—The quantity of matter in a body as indicated by its weight.

VI. *Unit of Mass.*—A body weighing *g lbs. g* being the rate of acceleration due to gravity.

VII. *Momentum or Dynamical Effect.*—The effect of a force with reference to its time of action.

VIII. *Centripetal Force.*—A force which tends to draw a body towards the centre of a circle in which it is constrained to move.

IX. *Centrifugal Force.*—The outward *reaction* tending from the centre of a circle in which a body is constrained to move.

X. *Vis viva of a particle.*—The product of the mass of a particle into the square of its velocity.

XI. *Centre of Oscillation.*—That point in a pendulum at which the whole mass might be collected without altering the time of oscillation.

XII. *Centre of Percussion.*—That point where an impulsive force may strike a body, having a fixed axis, without producing any pressure upon the axis.

DYNAMICAL PRINCIPLES.

1. *Parallelogram of Velocities.*—If two adjacent sides of a parallelogram represent the magnitude and direction of two velocities with which a particle is simultaneously animated, then the intermediate diagonal will represent the magnitude and direction of the resultant velocity.

2. *First Law of Motion.*—If a body is not acted upon by any force it will remain at rest; and if in motion, it will continue to move uniformly in the same direction.

3. *Second Law of Motion.*—If any number of forces act simultaneously upon a body at rest or in motion, each force produces the same effect in the direction of its action, as if it acted singly on the body at rest.

4. *Third Law of Motion.*—The velocity generated in any time by a force continually acting upon a body, is proportional to the force.

5. *Motion along an inclined plane or curve.*—The velocity acquired by a body acted upon by gravity whilst ascending or descending a smooth straight or curved line in a vertical plane, will be the same as if it ascended or descended freely through the same vertical space.

6. *Isochronous Chords.*—The time of descent down any chord of a circle drawn from either extremity of a vertical diameter, is equal to the time down that diameter.

7. *Conservation of the Motion of the Centre of Gravity.*—The common centre of gravity of any number of free particles acted upon by any forces, moves in the same manner as if the particles were concentrated in one body and acted upon by the same forces, in directions parallel to their actual directions.

8. *D'Alembert's Principle.*—When a system of material particles, rigidly or otherwise connected, is in motion under the action of any finite forces, the externally impressed moving forces will together with the *effective* moving forces on the *same* particles, applied in directions contrary to that in which they act, satisfy the conditions of statical equilibrium.

9. *Vis Viva.*—The vis viva of a connected system is, at any time, the same as if each particle, being free, had been acted upon by the same impressed forces through the same spaces.

10. *Laws of Gravitation.*—(1) The force of gravity on a body *exterior* to the earth's surface, is *inversely* proportional to the square of its distance from the earth's centre.

(2) The force of gravity on a body *within* the earth's surface, is *directly* proportional to its distance from the earth's centre.

DYNAMICAL FORMULÆ.

General Notation.

W = weight of a body.

M = mass of a body.

V = velocity of a body moving uniformly;
or the initial velocity of a projected body.

v = velocity acquired by a body moving with an accelerated or retarded motion.

s = space described by a body in time t.

f = rate of acceleration due to a pressure (P) continually applied to a body.

g = rate of acceleration due to gravity;
= 32·2 nearly, at Greenwich.

---

1. *Composition and Resolution of Velocities.*—Employ the statical formulæ, substituting *velocity* for *force*.

2. $$s = Vt.$$

3. $$v = ft.$$

4. $$s = \frac{1}{2}ft^2.$$

5. $$v^2 = 2fs.$$

6. $$v = V \pm ft.$$

7. $$s = Vt \pm \frac{1}{2}ft^2.$$

8. $$v^2 = V^2 \pm 2fS.$$

For falling bodies, $f = g = 32{\cdot}2$.

9. *For bodies projected vertically upwards, attaining the greatest height* (H) *in time* (T).

$$H = \frac{V^2}{64{\cdot}4}$$

$$T = \frac{V}{32{\cdot}2}.$$

10. $$M = \frac{W}{g}.$$

11. $$P = Mf.$$

12. Momentum $= Mv = Pt.$

13. $P : W = f : g.$ By the third law of motion.

PROJECTILES.

14. Equations of motion,

$$\left.\begin{aligned} x &= Vt \cos A \\ y &= Vt \sin A - \frac{1}{2} g t^2 \end{aligned}\right\} \quad \begin{aligned} &(1) \\ &(2) \end{aligned}$$

15. Equation to the parabolic path,

$$y = x \tan A - \frac{g x^2}{2 V^2 \cos^2 A}.$$

16. Horizontal range (R),

$$R = \frac{V^2}{g} \sin 2A.$$

17. Time of flight (T),

$$T = \frac{2V}{g} \sin A.$$

18. Greatest height (H),

$$H = \frac{V^2}{2g} \sin^2 A.$$

19. Latus rectum (L),

$$L = \frac{2V^2}{g} \cos^2 A.$$

20. Altitude of the directrix,

$$= \frac{V^2}{2g}.$$

21. Actual velocity at any point,

$$= \sqrt{V^2 - 2gy}.$$

22. Inclination to the horizon (θ) of the body's direction at any point,

$$\tan\theta = \tan A - \frac{g \sec^2 A}{V^2} x.$$

23. Velocity of a cannon ball (V),

$$V = 1600 \sqrt{\frac{3p}{w}}$$

where p, w are the weights of the powder and ball respectively.

SIMPLE PENDULUM.

24. (Length l inches : Time of oscillation T.)

$$T = \pi \sqrt{\frac{l}{g}}.$$

CENTRIFUGAL FORCE (F),

25. $$F = \frac{W}{g} \cdot \frac{V^2}{r}.$$

IMPACT ON A FIXED PLANE.

Angles of incidence and reflexion, A, θ.
Velocities before and after impact, V, v.
Modulus of elasticity, e.

26. $$\left.\begin{aligned} v \sin \theta &= V \sin A \\ v \cos \theta &= e V \cos A \end{aligned}\right\}$$

27. $$\tan \theta = \frac{1}{e} \tan A.$$

28. $$v = V \sqrt{\sin^2 A + e^2 \cos^2 A}.$$

COLLISION OF BODIES (M_1, M_2).

29. Whole momentum before impact = whole momentum after impact;

or.

$$M_1 V_1 + M_2 V_2 = M_1 v_1 + M_2 v_2 \qquad (1)$$

$$\frac{\text{Velocity of separation after impact}}{\text{Velocity of approach before impact}} = e,$$

or

$$\frac{v_2 - v_1}{V_1 - V_2} = e. \qquad (2)$$

30. $$v_1 = \frac{M_1 V_1 + M_2 V_2 - e M_2 (V_1 - V_2)}{M_1 + M_2}.$$

31. $$v_2 = \frac{M_1 V_1 + M_2 V_2 + e M_1 (V_1 - V_2)}{M_1 + M_2}.$$

HYDROSTATICS.

DEFINITIONS.

I. *Fluid.*—A body which can be divided in any direction, and its particles moved among each other by any force however small.

II. *Hydrostatics.*—The effects of force considered in reference to its action upon fluids at rest.

III. *Hydrodynamics.*—The effects of force considered in reference to its action upon fluids in motion.

IV. *Elastic Fluids.*—Fluids whose volume can be increased or diminished by the removal or application of external pressure.

V. *Non-Elastic Fluids.*—Fluids which undergo no sensible change of volume when the pressure upon them is increased or diminished.

VI. *Pressure at any point of a Fluid.*—The pressure which would be exerted on a rigid plane passing through that point.

VII. *Measure of Fluid pressure at any point.*—The pressure which the fluid would produce upon a unit of area if the pressure on every point of it were the same as at the proposed point.

VIII. *Density of a Substance.*—The mass of a unit of its volume.

IX. *Specific Gravity.*—The numerical ratio of the weights of equal volumes of any given substance and distilled water at temperature 60° Fahr.

X. *Plane of Floatation.*—The plane in which the surface of a fluid cuts a floating body.

XI. *Metacentre.*—The point in which a line drawn through the centres of gravity of a floating body and the fluid displaced by it, is intersected by a vertical line through the centre of gravity of the fluid displaced by the body after it has been made to revolve through a *very small* angle in a given vertical plane.

XII. *Centre of Pressure.*—That point of a plane surface immersed in fluid, where the *resultant* of the fluid pressure may be supposed to act.

HYDROSTATICAL PRINCIPLES.

1. Fluids press equally in all directions.

2. The pressure at any point in the interior of a fluid at rest is proportional to the depth of that point below the surface.

3. The surface of a fluid at rest is a horizontal plane.

4. If any number of vessels containing fluid at rest communicate with each other, the surfaces of the fluids in all the vessels will be at the same level.

5. The whole downward vertical pressure of a fluid on any surface is equal to the weight of the superincumbent volume of fluid.

6. *Hydrostatic Paradox.*—The pressure of a fluid on the horizontal base of the containing vessel depends *only* upon the area of the base and its vertical depth below the surface of the fluid, and *not* upon the form and volume of the containing vessel.

7. *Normal Pressure.*—The *whole* normal pressure of a fluid upon a surface immersed in it, is equal to the weight of a column of the fluid, having its base and height respectively equal to the area of the surface, and the depth of its centre of gravity below the surface of the fluid.

8. *Resultant Horizontal Pressure.*—The horizontal pressures upon the surface of a vessel containing fluid, or upon the surface of a body immersed in it, counteract each other ; or their *resultant* is equal to zero.

9. *Resultant Vertical Pressure.*—(1) The resultant of the vertical pressures of a fluid on a *body immersed* in it, is equal to the weight of the fluid displaced, and acts vertically upwards through its centre of gravity.

(2) The vertical pressure on any portion of the interior *surface of a vessel* containing fluid, is equal to the weight of the superincumbent volume of fluid.

10. *Conditions of Equilibrium of a floating body.*—The weight of the body is equal to that of the fluid displaced, and the line joining their centres of gravity is vertical.

11. *Conditions of Equilibrium of a body suspended in fluid.*—The difference between the *true* and *apparent* weights of the body is equal to that of the fluid displaced, and the centres of gravity of the body and the fluid displaced by it lie in the same vertical line with the point of suspension.

12. *Stable, Unstable, and Neutral Equilibrium.*—The equilibrium of a floating body is *stable*, *unstable*, or *neutral*, according as its centre of gravity is *below*, *above*, or coincident with the *metacentre*.

13. When two fluids meet in a bent tube, the altitudes of their surfaces above the horizontal plane in which they meet are inversely as their densities.

14. *Boyle's and Mariotte's Law.*—The pressure exerted by an elastic fluid at a constant temperature, is inversely as the space it occupies.

15. *Gay Lussac's and Dalton's Law.*—The pressure upon an elastic fluid being constant, the space it occupies is proportional to the temperature.

16. When a fluid, considered incompressible, flows through a tube of any form, and kept always full, the velocities of the fluid at any two points are inversely proportional to the areas of the transverse sections of the tube at those points.

17. The velocity with which a particle of incompressible fluid issues through an indefinitely small orifice in the containing vessel, is equal to that which it would have acquired by falling perpendicularly from the level of the fluid to that of the orifice.

FORMULÆ.

General Notation.

w = weight of a unit of volume of distilled water.
W = true weight of a solid in vacuo.
T = apparent weight of a solid in any fluid.
V = volume of a solid or fluid.
D = density of a solid or fluid.
S = specific gravity of a solid or fluid.
$\bar{z}$ = depth of the centre of gravity of a surface A immersed in fluid.

1. $$W = gDV.$$

2. $$W = wSV.$$

3. *Whole normal pressure on A,*

$$= wS \,.\, A \,.\, \bar{z}.$$

4. *Vertical downward pressure on A,*

= weight of the superincumbent volume of fluid.

5. *Resultant vertical pressure on a solid body volume V,*

$$= wS \,.\, V.$$

6. *Equilibrium of a floating body,*

$$W. = w \,.\, S \,.\, V.$$

7. *Equilibrium of a suspended body,*

$$W - T = w \,.\, S \,.\, V.$$

SPECIFIC GRAVITIES.

8. A solid heavier than water,

$$S = \frac{W}{W - T}.$$

9. A solid lighter than water,

$$S = \frac{W}{W'' - T'' - (W' - T')}.$$

10. Comparison of two fluids,

$$\frac{S}{S'} = \frac{W - T}{W - T'}.$$

11. Common hydrometer,

$$\frac{S}{S'} = \frac{V - a \cdot x'}{V - a \cdot x}.$$

12. Specific gravity of a compound,

$$= \frac{SV + S'V'}{V + V'}.$$

13. Specific gravity of air,

$$S = \frac{\text{flask of air} - \text{flask exhausted}}{\text{flask of water} - \text{flask exhausted}}.$$

The Barometer.

Pressure of the atmosphere (Π).
Weight of a cubic inch of mercury (m).
Difference of levels of the mercury (h).

14. $$\Pi = mh.$$

15. Difference of altitude measured in feet of two places,

$$= \frac{k}{g} \cdot \frac{\log \frac{h'}{h}}{\log e}.$$

Where $\sqrt{k} = 916{\cdot}2724$ nearly.

Air Pump.

16. Density of air in the receiver after n strokes,

$$D_n = D \cdot \left(\frac{V}{V+v}\right)^n.$$

CENTRE OF PRESSURE.

General Formulæ.

17. $$\bar{x} = \frac{\Sigma(ax^2)}{\Sigma(ax)} \text{ or } \frac{\iint x^2\,dx\,dy}{\iint x\,dx\,dy}.$$

$$\bar{y} = \frac{\Sigma(axy)}{\Sigma(ay)} \text{ or } \frac{\iint xy\,dx\,dy}{\iint x\,dx\,dy}.$$

The axis of y being the intersection of the area and the surface of the fluid. The axis of x in the plane of the area.

18. Rectangle, abscissæ of ends h, k,

$$\bar{x} = \frac{2}{3} \cdot \frac{k^3 - h^3}{k^2 - h^2}$$

19. Rectangle, just immersed,

$$\bar{x} = \frac{2}{3} \times \text{immersed side}.$$

20. Circular area, just immersed,

$$\bar{x} = \frac{5}{4} r.$$

EFFLUX OF FLUIDS

Through a very small orifice.

21. Velocity at orifice (v),

$$v = \sqrt{2 g z}.$$

22. Time of emptying (t),

$$t = -\int \frac{Z}{a \sqrt{2 g z}} dz.$$

Where a = area of the *effective* orifice or *vena contracta*

$$\doteq \frac{5}{8} \times \text{the orifice},$$

$Z \left(= f(z)\right)$ = area of the descending surface,

and z the depth of the orifice below the descending surface.

23. Cylindrical vessels,

$$t = \frac{2 \pi r^2}{a} \cdot \sqrt{\frac{h}{2g}}.$$

24. Conical vessel,

$$t = \frac{2 \pi r^2}{5 a} \cdot \sqrt{\frac{h}{2g}}.$$

25. Segment of a sphere,

$$t = \frac{\pi h}{a} \left(\frac{4}{3} r - \frac{2}{5} h\right) \sqrt{\frac{h}{2g}}.$$

REVETMENT WALLS.

DEF. *Revetment wall.*—A wall sustaining the pressure of earth or any loose material.

DEF. *Angle of natural slope.*—The inclination to the horizon, of the natural line of slope of a material.

Pressure on a revetment wall,

$$= w \tan^2 \left(45^\circ - \frac{\theta}{2} \right) . A\bar{z}.$$

Where w = weight of a cubic foot of the material; θ = angle of natural slope.

OPTICS.

DEFINITIONS.

I. *Medium.*—Any substance which allows the transmission of light.

II. *Ray.*—The smallest portion of light, considered as a straight line, which can proceed in any direction.

III. *Pencil of Rays.*—An assemblage of rays proceeding from a luminous point or origin of light.

IV. *Divergent or convergent Rays.*—Rays which meet in a point when produced in a direction *contrary to*, or *the same* as that of propagation *respectively*.

V. *Parallel Rays.*—Rays supposed to come from a point infinitely distant.

VI. *Angles of Incidence, Reflexion, and Refraction.*—The angles which a ray makes with the normal to the surface at the point of incidence before and after it is *reflected* or *refsacted*, respectively.

VII. *Critical Angle.*—The greatest angle of incidence which a ray can have in passing from a denser to rarer medium without being internally reflected.

VIII. *Virtual Ray.*—The *apparent* direction of a reflected ray.

IX. *Refractive Index.*—The ratio of the sines of the angles of incidence and refraction.

X. *Geometrical Focus.*—The limiting position of the points in which the rays of a reflected or refracted pencil cut the axis of the surface on which it is directly incident, when the breadth of the pencil is indefinitely diminished.

XI. *Principal Focus.*—The geometrical focus of a pencil of parallel rays.

XII. *Focal length of a Surface.*—The distance between the surface and the principal focus.

XIII. *Aberration.*—The distance between the point of intersection of a reflected or refracted ray with the axis of the surface, and the geometrical focus.

XIV. *Conjugate Foci.*—The *Origin* of a pencil of incident rays, and its *Geometrical Focus*—considered as convertible.

XV. *Deviation of a Ray.*—The angle between its direction after reflexion or refraction, and its original direction produced.

XVI. *Prism*—A portion of a refracting medium bounded by two plane surfaces terminating in a common line called the *edge of the prism*, and including an angle called the *refracting angle of the prism*.

XVII. *Lens.*—A portion of a refracting medium bounded by two spherical surfaces having a common axis called the *axis of the lens.*

XVIII. *Description of Lenses.*—Double convex; double concave; convexo-plane; concavo-plane; plano-convex; plano-concave; convexo-concave; concavo-convex. Light supposed to be incident on the first-named surface.

XIX. *Optical centre of a Lens.*—The point where a ray which suffers no deviation by refraction, cuts the axis of the lens.

XX. *Focal length of a Lens.*—The distance of its principal focus from the lens.

XXI. *Image of a Luminous Object.*—The locus of the geometrical foci of pencils of rays proceeding from the several points of the object; each focus supposed to be formed in the line drawn from the corresponding point of the object through the geometrical centre of the mirror, or through the optical centre of the lens. It is *Real* or *Virtual* according as it is formed by *Real* or *Virtual* rays. It is *Erect* or *Inverted* according as the object and its image *are* or *are not* on the same side of the axis of the mirror or lens. It is *Distorted* when the distances from the axis, of corresponding points in the object and its image are not in an invariable ratio.

XXII. *Long and Short Sight.*—Defects caused by too great *flatness* and *convexity* respectively, of the refracting surfaces of the eye.

XXIII. *Magnifying power of a Lens.*—The ratio of the visual angle of an image seen through the lens, to that of the object itself in the position of the image and viewed directly.

XXIV. *Magnifying power of a Telescope.*—The ratio of the visual angle of the image formed by the telescope, to that of the object viewed directly.

OPTICAL PRINCIPLES AND RESULTS.

1. Light falling upon the surface of a medium is *generally* divided into three parts, viz. (1) a part *irregularly dispersed* over the surface; (2) a part *regularly reflected;* (3) a part *regularly refracted.*

2. *Law of Reflexion.*—The angles of incidence and reflexion are equal; and the incident and reflected rays lie in the same plane with the normal at the point of incidence, and on opposite sides of it.

3. *Law of Refraction.*—The sines of the angles of incidence and refraction have the same ratio for the same kind of light and media; and the incident and refracted rays lie in the same plane with the normal at the point of incidence, and on opposite sides of it.

4. A pencil of *parallel* rays consists of *parallel* rays after reflexion or refraction at a plane surface. A pencil of *divergent* or *convergent* rays consists of *divergent* or *convergent* rays respectively, after reflexion or refraction at a plane surface.

5. A pencil of rays parallel to the axis of a paraboloid converges to the focus after reflexion at its surface. A pencil of rays diverging from or converging to a focus of a spheroid, will converge to or diverge from the other focus, after reflexion at its surface.

6. *Principle of Hadley's Sextant.*—The deviation of a ray after successive reflexions at two plane mirrors, is equal to twice the angle between the mirrors.

7. The axis of a pencil which passes through a prism or lens denser than the surrounding medium, is turned from the edge: and conversely.

8. If the refracting angle of a prism is greater than twice the critical angle of its substance, a pencil cannot pass through it, but is internally reflected.

9. A concave lens diminishes the convergency or increases the divergency of a pencil of rays.

10. A convex lens increases the convergency or diminishes the divergency of a pencil of rays.

11. *Formation of Images.*

Convex Mirror.—Image small, erect, behind the mirror, between it and the principal focus.

Concave Mirror.—(1) Image small, inverted, between the centre and principal focus, when the object is further from the mirror than its centre. (2) Image magnified, inverted, and further from the mirror than the centre, when the object is between the centre and principal focus. (3) Image magnified, erect, and behind the mirror, when the object is between the principal focus and the mirror.

Convex Lens.—(1) Image on the opposite side of the lens, inverted, and at distance greater than the focal length, when the distance of the object from the lens is greater than its focal length. (2) Image erect, and on the same side of the lens as the object, when the object is between the lens and its principal focus.

Concave Lens.—Image small, erect, on the same side of the lens as the object, and between the lens and its principal focus.

12. *Defective sight, its remedies.*—Short sight requires a concave lens; long sight a convex lens.

13. *Dispersion of light. Prismatic colours.*

Newton's Spectrum.

(Least refracted) Red.
Orange.
Yellow.
Green.
Blue.
Indigo.
(Most refracted) Violet.

Brewster's Spectrum.

Primary Colours.	Secondary Colours.
Red.	Orange.
Yellow.	Green.
Blue.	Violet.

OPTICAL FORMULÆ.

General Notation.

A = angle of incidence.

θ = angle of refraction.

μ = the refractive index.

r = radius of a reflecting or refracting surface.

u = distance of the origin of light from the surface.

v = distance of the geometrical focus " "

(u, v positive in direction contrary to that of incident light).

f = the distance of the principal focus from the surface; and the focal length of a lens.

t = thickness of a plate or lens.

I = illumination at any point of a surface, at a distance (d) from the origin of light.

1. $I = C \cdot \dfrac{\cos A}{d^2}$. ($C$ a constant).

2. $\mu = \dfrac{\sin A}{\sin \theta}$.

Reflexion.

Plane surface. Direct or oblique incidence.

3. $$v = -u.$$

Spherical surface. Direct incidence.

4. $$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

5. $$f = \frac{r}{2}.$$

6. $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

7. $$(u - f)(v - f) = f^2.$$

8. Aberration $= v' - v$.

$$v' - v = -\frac{\left(\frac{1}{r} - \frac{1}{u}\right)^2}{\left(\frac{2}{r} - \frac{1}{u}\right)^2} \cdot \frac{y^2}{r}.$$

Parabolic surface.

9. $$\frac{1}{v} + \frac{1}{u} = \frac{1}{a}.$$

Spherical surface. Oblique incidence.

10. $$\begin{cases} \dfrac{1}{v_1} + \dfrac{1}{u} = \dfrac{2}{r \cos \Delta}. & (1) \\[2ex] \dfrac{1}{v_2} + \dfrac{1}{u} = \dfrac{2 \cos \Delta}{r}. & (2) \end{cases}$$

Refraction.

Plane surface. Direct incidence.

11. $$v = \mu \cdot u.$$

Plane surface. Oblique incidence.

12.
$$\begin{cases} \dfrac{\mu \cos^2 \theta}{v_1} - \dfrac{\cos^2 A}{u} = 0. & (1) \\[2ex] \dfrac{\mu}{v_2} - \dfrac{1}{u} = 0. & (2) \end{cases}$$

Spherical surface. Direct incidence.

13.
$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}.$$

14.
$$f = \frac{\mu}{\mu - 1} \cdot r.$$

15. Aberration $= v' - v$.

$$v' - v = -\frac{\mu - 1}{\mu} \frac{\left(\dfrac{1}{r} - \dfrac{1}{u}\right)^2}{\left(\dfrac{\mu - 1}{r} + \dfrac{1}{u}\right)^2} \left(\frac{1}{r} - \frac{\mu + 1}{u}\right) \frac{y^2}{2}.$$

Spherical surface. Oblique incidence.

16.
$$\begin{cases} \dfrac{\mu \cos^2 \theta}{v_1} - \dfrac{\cos^2 A}{u} = \dfrac{\mu \cos \theta - \cos A}{r}. & (1) \\[2ex] \dfrac{\mu}{v_2} - \dfrac{1}{u} = \dfrac{\mu \cos \theta - \cos A}{r}. & (2) \end{cases}$$

Combined Refractions.

17. Critical angle $= \sin^{-1} \dfrac{1}{\mu}$.

Refraction through n parallel media.

18.
$$\mu = \mu_1 \cdot \mu_2 \cdot \mu_3 \cdots \mu_{n-1}.$$

Plate.

19.
$$v = u + \frac{t}{\mu}.$$

20 $$\text{Aberration} = v' - v.$$

$$v' - v = -\frac{\mu^2 - 1}{\mu^3} \cdot \frac{t}{u^2} \cdot \frac{y^2}{2}.$$

Plate. Oblique incidence.

21. $$\begin{cases} v_1 = u + t \cdot \dfrac{\cos^2 A}{\mu \cos^3 \theta}. & (1) \\ v_2 = u + \dfrac{t}{\mu \cos \theta}. & (2) \end{cases}$$

Prism. (Refracting angle $= i$.)

22. $$\begin{cases} i = A' \pm \theta. & (1) \\ \sin A = \mu \sin \theta. & (2) \\ \sin \theta' = \mu \sin A'. & (3) \\ \text{Deviation} = \theta' - A' \pm (A - \theta.) & (4) \end{cases}$$

23. $$\begin{cases} v_1 = \dfrac{\cos^2 \theta}{\cos^2 A} \cdot \dfrac{\cos^2 \theta'}{\cos^2 A'} \cdot u. & (1) \\ v_2 = u. & (2) \end{cases}$$

Lens. Direct refraction.

24. $$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) - \frac{t}{\mu}\left(\frac{1}{u} + \frac{\mu - 1}{r}\right)^2.$$

25. $$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right). \quad \text{If } t = 0.$$

Or,

26. $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

27. $$\text{Aberration} = v' - v.$$

$$= -\frac{\mu-1}{\mu^2}\left\{\left(\frac{1}{r}-\frac{1}{u}\right)^2\left(\frac{1}{r}-\frac{\mu+1}{u}\right) - \left(\frac{1}{s}-\frac{1}{v}\right)^2\left(\frac{1}{s}-\frac{\mu+1}{v}\right)\right\}\frac{v^2y^2}{2}.$$

Lens. Oblique refraction.

28. $$\left\{\begin{aligned} &\frac{1}{v_1}-\frac{1}{u}=\frac{\mu\cos\theta-\cos A}{\cos^2 A}\left(\frac{1}{r}-\frac{1}{s}\right). && (1)\\ &\frac{1}{v_2}-\frac{1}{u}=(\mu\cos\theta-\cos A)\left(\frac{1}{r}-\frac{1}{s}\right). && (2)\end{aligned}\right.$$

Sphere. Direct refraction.

29. $$\frac{1}{v}-\frac{1}{u}=-2\frac{\mu-1}{\mu r}=\frac{1}{f},$$

r, u, f measured from the centre of the sphere.

Combined Lenses.

30. $$\left\{\begin{aligned} &\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_1}.\\ &\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\ &\frac{1}{v_n}-\frac{1}{v_{n-1}+a_{n-1}}=\frac{1}{f_n}.\end{aligned}\right.$$

31. $$\frac{1}{v_n}-\frac{1}{u}=\Sigma\left(\frac{1}{f}\right).\quad\text{If } t=0.$$

Condition of Achromatism.

32. $$0=\Sigma\left(\frac{\delta\mu}{\mu-1}\cdot\frac{1}{f}\right).$$

Determination of the Focal Length of a Lens by experiment.

33. Convex Lens,

$$\begin{cases} v + u = \text{minimum} & (1) \\ f = \dfrac{v+u}{4}. & (2) \end{cases}$$

34. Concave Lens,

$$\frac{1}{f} = \frac{1}{F} - \frac{1}{f'}.$$

Astronomical Telescope.

35. Magnifying power $= \dfrac{\text{focal length of object glass}}{\text{focal length of eye glass}}$.

ASTRONOMY.

DEFINITIONS.

I. *Celestial Sphere.*—The *imaginary* spherical surface by which space is *apparently* bounded, the eye of the spectator being the centre.

II. *Secondary Circles.*—Great circles perpendicular to the plane of a given circle.

III. *Zenith and Nadir.*—The points in which a plumb-line at any place, produced indefinitely, may be supposed to meet the celestial sphere above and below that place respectively.

IV. *North and South Poles.*—The points in which the earth's axis produced both ways meets the celestial sphere.

V. *Visible Horizon of a place on the Earth's Surface.*—The boundary of view at that place, as determined by tangents to the earth's surface, passing through the eye of the spectator.

VI. *Dip of the Horizon.*—The angle of depression of the visible horizon.

VII. *Sensible Horizon.*—A plane touching the earth's surface, and bounded by the celestial sphere.

VIII. *Rational Horizon.*—A plane passing through the earth's centre and parallel to the sensible horizon.

IX. *Terrestrial Equator.*—The great circle on the earth's surface made a plane through the centre of the earth, and perpendicular to its axis.

X. *Celestial Equator.*—A great circle of the celestial sphere made by the plane of the *Terrestrial Equator* extended indefinitely.

XI. *Vertical Circles.*—Circles whose planes pass through the zenith and nadir.

XII. *Declination Circles.*—Great circles of the celestial sphere whose planes pass through the north and south poles.

XIII. *Celestial Meridian of a place.*—A great circle passing through the poles and the zenith.

XIV. *Prime Vertical.*—The vertical circle whose plane is perpendicular to that of the meridian.

XV. *Terrestrial Meridian of a place.*—The great circle in which the celestial meridian cuts the earth's surface.

XVI. *Ecliptic.*—The section of the celestial sphere made by the plane of the sun's apparent path.

XVII. *Obliquity of the Ecliptic.*—The angle of inclination of the ecliptic and equator (23° 28′ nearly).

XVIII. *Equinoxes: Vernal and Autumnal.*—The points in which the equator and ecliptic intersect.

XIX. *First point of Aries.*—The vernal equinox.

XX. *Right Ascension and Declination of a Heavenly Body.*—Its spherical co-ordinates referred to the equator and the first point of Aries.

XXI. *Polar Distance.*—The complement of the declination.

XXII. *Hour Angle of a Heavenly Body.*—The angle between the meridian of a place and a declination circle passing through the body.

XXIII. *Azimuth and Altitude of a Heavenly Body.*—Its spherical co-ordinates referred to the rational horizon and that point in it most remote from the elevated pole.

XXIV. *Solstitial Colure.*—The great circle through the poles of the ecliptic and equator.

XXV. *Solstices: Summer and Winter.*—The points in which the solstitial colure intersects the ecliptic.

XXVI. *Latitude and Longitude of a Heavenly Body.*—Its spherical co-ordinates referred to the ecliptic and the first point of Aries.

XXVII. *Angle of Position of a Heavenly Body.*—The angle between its circles of declination and latitude.

XXVIII. *Perihelion or Perigee (according as the* EARTH *or* SUN *is considered the Moving Body).*—That point in the orbit at which the distance between them is least.

XXIX. *Aphelion or Apogee.*—That point in the orbit at which the distance between them is the greatest.

XXX. *Line of Apsides.*—The line joining the *apsides*, or *aphelion* and *perihelion*.

XXXI. *Solar, Lunar, Sidereal Day.*—The interval between two successive transits of the meridian of a place over the *sun*, *moon*, or a fixed *star* respectively.

XXXII. *Mean Solar Time.*—Time reckoned by the position of a fictitious sun moving in the equator with the true sun's mean motion in longitude.

XXXIII. *Equation of the Centre.*—The difference between the *true* and *mean* longitude of the earth.

XXXIV. *Equation of Time.*—The difference between mean and apparent time, at noon.

XXXV. *Precession of the Equinoxes.*—A slow change in their position, arising from the attraction of the Sun and Moon upon the protuberant matter at the earth's equator.

XXXVI. *Nutation of the Earth's Axis.*—A slow change in its inclination to the axis of the ecliptic arising from the same cause which produces precession.

XXXVII. *Parallax.*—The angle subtended at an object by the line which joins two different points from which it may be viewed.

ASTRONOMICAL RESULTS AND LAWS.

1. The motions of all the planets in their orbits are from *west* to *east*, and are all subject to Kepler's Laws.

2. *Kepler's Laws.*—(1) The radii vectores describe areas in one plane proportional to the time. (2) The orbits are ellipses having the sun in one of their foci. (3) The squares of the periodic times are proportional to the cubes of the mean distances from the Sun.

3. *Deductions from Kepler's Laws.*

From Law (1)—the force which acts upon the bodies tends always to the centre of the sun. From Law (2)—the force varies inversely as the square of the distance. From Law (3)—the absolute force is the same for all the planets.

4. *Bode's Law.*—Vide Appendix, Table XVI.

ASTRONOMICAL FORMULÆ.

Notation.

l, c denote the latitude and colatitude of a place.

δ, p „ the declination and polar distance of a celestial body.

z and z' „ the true and apparent zenith distances.

A or a and A' or a' „ „ altitudes.

h „ hour angle.

1. Distance (D miles) of the *offing* or *visible horizon* from the eye, height (k feet); And Dip (a minutes),

$$D = \sqrt{\frac{3}{2}k} \text{ approximately,}$$

$$a = \cdot 9784\sqrt{k} \quad \text{„}$$

2. Latitude by the sun's meridian altitude,

$$l = \delta + z.$$

3. Latitude by a circumpolar star,

$$l = \frac{A + a}{2}.$$

4. Latitude by the pole star out of the meridian,

$$l = a - p\cos h + \frac{1}{2}\tan a\,(p\sin h)^2 \sin 1''.$$

5. Hour angle, by an observed altitude:

$$\sin\frac{h}{2} = \sqrt{\frac{\sin\frac{z+p-c}{2}\sin\frac{z+c-p}{2}}{\sin c\sin p}}.$$

6. Length of day ($2t$),

$$\cos(t \times 15°) = -\tan l \,.\, \tan \delta.$$

7. Refraction,

$$= 57'' \tan z' \text{ nearly.}$$

8. Parallax in altitude,

$$= \frac{r}{R} \,.\, \sin z'.$$

9. Horizontal parallax,

$$= \frac{r}{R}.$$

Where r = earth's radius, R = earth's distance from the body.

10. Longitude by the chronometer,

= Greenwich time $\sim$ corrected time at any place.

Longitude by lunar distances,

11.
(1) $\sin^2 \frac{D}{2} = \cos\left(\frac{A+a}{2} + \theta\right) \cos\left(\frac{A+a}{2} - \theta\right).$

(2) $\sin^2 \theta = \sec a' \,.\, \sec A' \cos a \cos A \cos X \cos(X - D').$

(3) $X = \frac{1}{2}(A' + a' + D').$

Where D, D' denote the true and apparent distances respectively.

HEAT.

EXPLANATION OF TERMS.

I. *Temperature.*—The energy with which the heat in a body tends to transfer itself to other bodies.

II. *Sensible Heat.*—Heat sensibly employed in raising matter to a given temperature.

III. *Latent Heat.*—Heat which disappears in changing the state of matter from solid to liquid, or from liquid to vapour.

IV. *Unit of Heat.*—The quantity of heat required to raise 1lb. of water through 1° Fahr.

V. *Mechanical Measure of the Unit of Heat between limits of Temperature* 55° *and* 60° = 772 *Units of Work.*

VI. *Specific Heat.*—The ratio of the quantities of heat required to raise equal weights of any proposed substance and water through 1° Fahr.

VII. *Conduction.*—The process by which heat is transmitted from one portion of matter to another through the interior of an intermediate body.

VIII. *Radiation.*—The process by which heat is transmitted from one body to another by rays through an intermediate space.

IX. *Convection.*—The process by which the temperature of liquids and gases is raised, by the circulation of particles rendered specifically lighter when heated.

X. *Dew Point.*—That temperature at which the vapour with which the atmosphere is charged would begin to deposit itself in the form of dew.

XI. *Thermometers.*

Standard Readings.	Fahrenheit. F.	Centigrade. C.	Réaumur. R.
Boiling Point.	212°	100°	80°
Freezing Point	32	0	0

$\sin$

$\sin$

$\cos$

$\cos$

$\tan$

$\tan$

$\sin A +$

$\sin A -$

$\cos A +$

$\cos B -$

n a *lineal*
s caused
ationary

rations
ty as to

lowest
musical

e of a

sounded
have a
is not

ng of 7

Octave.
C
2
1
2/1

Formulæ for Comparison.

$$\frac{F^\circ - 32^\circ}{9} = \frac{C^\circ}{5} = \frac{R^\circ}{4}.$$

XII. *Expansion or Contraction by Heat.*

Linear expansion $= e\,(t' - t)\,l$.

Where e = expansion for 1° of temperature. l the length at temperature t.

Cubic expansion = 3 × linear expansion, *nearly*.

ACOUSTICS.

DEFINITIONS AND PRINCIPLES.

1. *Sound.*—The effect produced upon the organs of hearing by the vibrations of a body, transmitted to the ear by some conducting medium.

2. *Conditions for the effective transmission of sound.*—The conducting medium should possess *elasticity*, and be of *uniform density*.

3. *Velocity of Sound*, at t° Fahr.

$$v = V\sqrt{\frac{t + 461{\cdot}2}{493{\cdot}2}}.$$

Where V = velocity in pure dry air at 32° Fahr.
= 1090·2 by calculation.
$= \left.\begin{matrix}1090{\cdot}1\\1090{\cdot}5\end{matrix}\right\}$ by experiment.

4. *Nodal Points and Lines.*—Those *points* or *lines* in a *lineal* or *superficial* body respectively, where the undulations caused by opposite vibrations meet each other, causing a stationary *point* or *line.*

5. *Musical Sound.*—The effect of a series of vibrations succeeding each other with such rapidity and regularity as to produce the impression of a *single* sound.

6. *Musical Interval.*—The ratio, expressed in its lowest terms, of the number of vibrations made by two musical notes in the same time.

7. *Pitch of a Note.*—Varies inversely as the time of a vibration.

8. *Concord.*—The effect produced by two notes sounded together, their vibrations being so related as to have a common period after a few oscillations. Where this is not the case *Discord* is produced.

9. *Diatonic and Chromatic Scales.*—Scales consisting of 7 and 12 intervals respectively.

DIATONIC SCALE.

Number of vibrations of	Fund.	Second.	Third.	Fourth.	Fifth.	Sixth.	Seventh.	Octave.
	C	D	E	F	G	A	B	C
Upper Note . .		9	5	4	3	5	15	2
Lower Note . .	1	8	4	3	2	3	8	1
Intervals . . .		$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	$\frac{2}{1}$

APPENDIX.

TABLE I.

Square, Cubes, Square Roots, Cube Roots, Reciprocals.

Num.	Square.	Cube.	Square Root.	Cube Root.	Reciprocals.
1	1	1	1·00000	1·00000	1·00000
2	4	8	1·41421	1·25992	0·50000
3	9	27	1·73205	1·44225	·33333
4	16	64	2·00000	1·58740	·25000
5	25	125	2·23607	1·70998	·20000
6	36	216	2·44949	1·81712	·16667
7	49	343	2·64575	1·91293	·14286
8	64	512	2·82843	2·00000	·12500
9	81	729	3·00000	2·08008	·11111
10	100	1000	3·16228	2·15443	·10000
11	121	1331	3·31662	2·22398	·09091
12	144	1728	3·46410	2·28943	·08333
13	169	2197	3·60555	2·35133	·07692
14	196	2744	3·74166	2·41014	·07143
15	225	3375	3·87298	2·46621	·06667
16	256	4096	4·00000	2·51984	·06250
17	289	4913	4·12311	2·57128	·05882
18	324	5832	4·24264	2·62074	·05556
19	361	6859	4·35890	2·66840	·05263
20	400	8000	4·47214	2·71442	·05000
21	441	9261	4·58258	2·75892	·04762
22	484	10648	4·69042	2·80204	·04545
23	529	12167	4·79583	2·84387	·04348
24	576	13824	4·89898	2·88450	·04167
25	625	15625	5·00000	2·92402	·04000
26	676	17576	5·09902	2·96250	·03846
27	729	19683	5·19615	3·00000	·03704
28	784	21952	5·29150	3·03659	·03571
29	841	24389	5·38516	3·07232	·03448
30	900	27000	5·47723	3·10723	·03333

TABLE II.

Prime Numbers and their Logarithms.

Num.	Logarithm.	Num.	Logarithm.
0	— ∞	113	2·05308
1	0·00000	127	2·10380
3	0·47712	131	2·11727
5	0·69897	137	2·13672
7	0·84510	139	2·14301
11	1·04139	149	2·17319
13	1·11394	151	2·17898
17	1·23045	157	2·19590
19	1·27875	163	2·21219
23	1·36173	167	2·22272
29	1·46240	173	2·23805
31	1·49136	179	2·25285
37	1·56820	181	2·25768
41	1·61278	191	2·28103
43	1·63347	193	2·28556
47	1·67210	197	2·29447
53	1·72428	199	2·29885
59	1·77085	211	2·32428
61	1·78533	223	2·34830
67	1·82607	227	2·35603
71	1·85126	229	2·35984
73	1·86332	233	2·36736
79	1·89763	239	2·37840
83	1·91908	241	2·38202
89	1·94939	251	2·39967
97	1·98677	257	2·40993
101	2·00432	263	2·41996
103	2·01284	269	2·42975
107	2·02938	271	2·43297
109	2·03743	277	2·44248

TABLE III.

Lineal Measure.

12 inches	=	1 foot.
3 feet	=	1 yard.
$5\frac{1}{2}$ yards	=	1 perch or pole.
40 perches	=	1 furlong.
8 furlongs	=	1 mile.
3 miles	=	1 league.

---

1 mile	=	1760 yards = 5280 feet.
1 fathom	=	6 feet.
1 knot	=	1000 fathoms.
1 degree	=	$69\frac{1}{2}$ miles.
1 chain	=	100 links.
,,	=	22 yards.

Superficial Measure.

144	square	inches	=	1	square	foot.
9	,,	feet	=	1	,,	yard.
$30\frac{1}{4}$	,,	yards	=	1	,,	pole.
40	,,	poles	=	1	,,	rood.
4	,,	roods	=	1	,,	acre.

---

1 square acre	=	100000	square	links.
,,	=	4840	,,	yards.
,,	=	10	,,	chains.
1 square mile	=	640	,,	acres.

TABLE IV.

Comparison of French and English Weights and Measures.

	French.		English.
Weights	Gramme	=	15·438 grains troy.
	Kilogramme	=	2·68027 lbs. „
Lineal Measure.	Millimètre	=	0·03937 inches.
	Centimètre	=	0·393708 „
	Decimètre	=	3·937079 „
	Mètre	=	3·2808992 feet.
	Myriamètre	=	6·2138 miles.
Superficial Measure.	Mètre carré	=	1·196033 square yards.
	Are	=	0·098845 „ roods.
	Hectare	=	2·471143 „ acres.
Measure of Capacity.	Litre	=	1·760773 pints.
	Decalitre	=	2·2009668 gallons.
	Hectolitre	=	22·009668 „

TABLE V.

Circular Measures and Natural Trigonometrical Ratios.

Angle.	Circular Measures.	Sines.	Tangents.	Co-tangents.	Cosines.	Angle.
1′	·000291	·000291	·000291			
1°	·017453	·017452	·017455	57·2900	·999848	89°
3	0·0524	0·0523	0·0524	19·0811	0·9986	87
5	·0873	·0872	·0875	11·4300	·9962	85
7	·1221	·1219	·1228	8·1443	·9925	83
9	·1571	·1564	·1584	6·3138	·9877	81
11	·1920	·1908	·1944	5·1416	·9816	79
13	·2269	·2250	·2309	4·3315	·9744	77
15	·2618	·2588	·2679	3·7321	·9659	75
17	·2967	·2924	·3057	3·2709	·9563	73
19	·3316	·3256	·3443	2·9042	·9455	71
21	·3665	·3584	·3839	2·6051	·9336	69
23	·4014	·3907	·4245	2·3559	·9205	67
25	·4363	·4226	·4663	2·1445	·9003	65
27	·4712	·4540	·5095	1·9626	·8910	63
29	·5061	·4848	·5543	1·8040	·8746	61
31	·5410	·5150	·6009	1·6643	·8572	59
33	·5760	·5446	·6494	1·5399	·8387	57
35	·6109	·5736	·7002	1·4281	·8192	55
37	·6458	·6018	·7536	1·3270	·7986	53
39	·6807	·6293	·8098	1·2349	·7771	51
41	·7156	·6561	·8693	1·1504	·7547	49
43	·7505	·6820	·9325	1·0724	·7314	47
45	·7854	·7071	1·0000	1·0000	·7071	45
		Cosine.	Co-tangent.	Tangent.	Sine.	Angle.

TABLE VI.

Compound Interest and Annuities. 5 *per Cent.*

Years (n).	Amount of £1 in (n) years.	Amount of £1 per Annum in (n) years.	Present Value of £1 per Annum for (n) years.
	£	£	£
5	1·2763	5·5256	4·3295
10	1·6289	12·5779	7·7217
15	2·0789	21·5786	10·3797
20	2·6533	33·0660	12·4622
25	3·3864	47·7271	14·0939
30	4·3219	66·4388	15·3725
35	5·5160	90·3203	16·3742
40	7·0400	120·7998	17·1591
45	8·9850	159·7002	17·7741
50	11·4674	209·3480	18·2559
55	14·6356	272·7126	18·6335
60	18·6792	353·5837	18·9293
65	23·8399	456·7980	19·1611
70	30·4264	588·5285	19·3427
75	38·8327	756·6537	19·4850
80	49·5614	971·2288	19·5965
85	63·2544	1245·0871	19·6838
90	80·7304	1594·6073	19·7523
95	103·0347	2040·6935	19·8059
100	131·5013	2610·0252	19·8479
Perp^{y}.	—	—	20·0000

TABLE VII.

Annual Premiums for Assuring £100 at Death, and Present Values of a Life Annuity of £100 per Annum.

Age.	Annual Premiums.	Present Value of Annuities.	Expectation of Life.
	£	£	Years.
20	1·689	1706·75	
25	1·818	1665·58	37·805
30	2·023	1603·92	33·681
35	2·381	1505·79	29·721
40	2·828	1398·22	25·944
45	3·450	1270·67	22·365
50	4·260	1133·76	18·994
55	5·240	1000·52	15·832
60	6·527	864·00	12·878
65	8·417	715·48	10·264
70	10·867	579·66	8·113
75	13·724	469·17	6·345
80	16·854	383·08	4·884

From the experience of "The Amicable" Insurance Office.

TABLE VIII.

Work of Living Agents.

Agent.	No. of Units of Work per Minute.
A MAN transporting his own weight horizontally	42330
,, walking with a weight on his back, or transporting material in a wheelbarrow	13030
,, raising his own weight	4230
,, walking and pushing or pulling horizontally, or rowing . . .	3130
,, turning a winch	2600
,, raising water from a well with a windlass	2560
,, pulling and pushing alternately in a vertical direction	2390
,, working a pump	1980
,, raising weights with a pulley . . .	1560
,, raising weights by hand	1480
,, carrying weights up a ladder . . .	1130
,, raising water with rope and bucket	1054
,, wheeling material up an incline of 1 in 12	520
,, lifting earth with a spade $5\frac{1}{4}$ feet .	470
A HORSE trotting with a weight on his back	76410
,, walking with a weight on his back	57310
,, yoked to a cart and walking . .	26150

TABLE IX.

Specific Gravities, and Absolute Weights in Pounds per Cubic Foot.

Substance.	Specific Gravity.	Weight.
Platinum	20·980	1311·25
Gold	19·307	1206·69
Mercury (at 32° F.)	13·619	851·18
Lead	11·446	717·45
Silver	10·312	644·50
Copper	8·607	537·93
Brass	8·399	525·00
Steel (Soft)	7·780	486·25
Tin	7·291	455·68
Iron (Cast)	7·113	444·56
Zinc	7·028	439·25
Diamond	3·521	220·06
Stone (White Flint)	2·594	162·13
Glass (Plate)	2·453	153·31
Clay (Common)	1·919	119·93
Sand (Damp)	1·886	117·87
Ivory	1·826	114·12
Sand (Dry)	1·420	88·60
Coal	1·257	78·56
Milk	1·032	64·55
Water (Sea)	1·027	64·18
Water (Rain)	1·000	62·50
Oil (Linseed)	0·940	58·75
Oak	0·934	58·37
Spirits of Wine	0·829	51·82
Deal (English)	0·470	29·37
Cork	0·240	15·00
Air	0·00123	0·0768
Hydrogen	0·00009	0·0056

TABLE X.

Coefficients of Friction and Angles of Repose.

Material.	Coefficient of Friction.	Angle of Repose.
Shingle and Gravel on Ditto	0·81 to 1·11	39° to 48°
Damp Clay on Ditto . .	1·00	45
Dry Sand on Ditto . .	0·38 to 0·75	21 to 37
Masonry and Brickwork with damp Mortar . .	0·74	36·5
Dry Masonry and Brickwork	0·60 to 0·70	31 to 35
Iron on Stone	0·30 to 0·70	16·7 to 35
Timber on Metals . . .	0·20 to 0·60	11·3 to 31
Masonry on dry Clay . .	0·51	27
Timber on Timber . .	0·20 to 0·50	11·3 to 26·5
Timber on Stone . . .	0·40	22
Masonry on moist Clay .	0·33	18·3
Wet Clay on Ditto . .	0·31	17
Metals on Metals . . .	0·15 to 0·25	8·5 to 14

TABLE XI.

Strength of Materials.

Material.	Weight of Cubic foot in lbs. Av.	Modulus of Elasticity.	Coefficients of Absolute Strength, or Rupture in lbs. Av. per square inch.		
			Extension.	Compression.	Transverse Strain.
Ash	48·5	1644800	17207	9023	12156
Beech . . .	48·2	1353600	16817	9363	9336
Box	60·0		19891	10299	
Brass (cast) .	525·0	8930000	17968	10304	
Brass (wire) .		14230000	49000		
Brick . . .	135·5		280	807	
Cedar . . .	56·8		11400	5674	7400
Copper (cast).	537·9		19072		
Copper (wire)		17000000	60000		
Deal (Christiana)	43·6	1672000	12400		9864
Elm	36·8	699840	13489	10331	6078
Fir (Riga) . .	47·1	1328800	11549	5748	6648
Glass . . .	153·3	8000000	9420		
Iron (cast). .	444·6	17000000	16500	112000	40000
Iron (wire) .		25300000	85000		
Iron (wrot.) .	481·2	29000000	65000	38000	42000
Ivory . . .	114·1		16626		
Larch . . .	32·6	897600	10220	3201	4992
Lead (cast) .	715·5	720000	1824		
Lignum Vitæ.	76·3		11800	9900	12000
Mahogany. .	50·0	1255000	14900	8200	7600
Mortar. . .	107·2		50		
Oak	58·4	1451200	17300	9509	10032
Pine (red). .	41·1	1680000	13000	5788	8320
Rope (hempen)	*		5600		
Sandstone. .	144·0			3850	1730
Slate . . .	180·5	15800000	12800		11766
Steel . . .	490·0	35500000	115000		
Teak . . .	41·1	2414400	15000	12101	14772
Tin	455·7	4608000	5322		
Walnut. . .	41·9		8130	6645	
Yew. . . .	50·4		8000		

* Weight of Hempen Rope 1 foot long, 1 inch circumference = 0·045 lb.

TABLE XII.

Ratios of Coefficients of Safety and Ultimate Strength.

Ordinary Steel and Wrought Iron	$\frac{1}{5}$
Wrought Iron Boilers	$\frac{1}{8}$
Cast Iron (steady load)	$\frac{1}{4}$
„ (moving load)	$\frac{1}{8}$
Timber (average)	$\frac{1}{10}$
Stone and Brick (average)	$\frac{1}{8}$

TABLE XIII.

Linear Expansion of Materials for 1° *Fahr.*

Zinc	=	0·00001634
Lead	=	0·00001592
Brass Rods	=	0·00001052
Copper	=	0·00000944
Iron (wrought)	=	0·00000642
Steel Rods	=	0·00000636
Iron (cast)	=	0·00000617
Platinum	=	0·00000476
Glass Tube	=	0·00000431

Mercury (in Glass) = 0·00008696

TABLE XIV.

Elasticity of Bodies in Collision.

Substance.	Elasticity (e).
Glass . . .	·94
Ivory . . .	·89
Steel	·79
India Rubber .	·67
Cast Iron . .	·66
Cork	·65
Elm	·60
Bell Metal . .	·59
Lead *versus* Cork . . .	·57
Brass *v.* Steel .	·52
Stone . . .	·39
Brass . . .	·36
Lead	·20
Lead *v.* Steel .	·19
Clay (Moist) .	·17

TABLE XV.

Refractive Indices.

Substance.	Refractive Index.
Chromate of Lead	2·974
Diamond	2·450
Sulphur	2·115
Garnet	1·815
Ruby	1·779
Tourmaline . . .	1·688
Flint Glass . . .	1·616
Emerald	1·585
Crown Glass . . .	1·535
Plate Glass . . .	1·525
Oil of Olives . .	1·470
Fluor Spar . . .	1·436
Alcohol	1·372
Salt Water . . .	1·343
Distilled Water .	1·336
Ice	1·307
Air	1·000294
Oxygen	1·000272
Hydrogen . . .	1·000138

TABLE XVI.

The Planetary System.

Name of Body.	Mean Distance from the Sun.	Bode's *Empirical* Law of Planetary Distances.	Periodic Time in Mean Solar Days.	Diameter in Miles.	Density.
Sun . .	. . .	. . . . . .	. . .	882000	0·25
Mercury	0·3870981	$4=4$	87·969	3140	1·12
Venus .	0·7233316	$7=4+3\times1$	224·701	7800	0·92
Earth .	1·0000000	$10=4+3\times2$	365·256	7926	1·00
Mars .	1·5236923	$16=4+3\times2^2$	686·980	4100	0·95
Flora .	2·2016870		1193·249		
Vesta .	2·3610810		1325·147	250?	
Iris . .	2·3806240		1341·636		
Metis .	2·3856070		1345·850		
Hebe .	2·4257866	$28=4+3\times2^3$	1379·994		
Astræa .	2·5770470		1511·095		
Juno .	2·6708370		1594·296	79?	
Ceres .	2·7680510		1682·125	163?	
Pallas .	2·7728580		1686·510		
Jupiter .	5·2027760	$52=4+3\times2^4$	4332·585	87000	0·24
Saturn .	9·5387861	$100=4+3\times2^5$	10759·220	79160	0·14
Uranus .	19·1823900	$196=4+3\times2^6$	30686·821	34500	0·24
Neptune	30·0368000	?	60126·710	41500	0·14

TABLE XVII.

Miscellaneous Data.

1.	1 lb. Avoirdupois	= 7000 grains.
2.	1 lb. Troy	= 5760 grains.
3.	$\log_{10} \pi$	= 0·49715.
4.	$\log_{10}$ (unit of circular measure)	= 1·75812.
5.	$\log_{10} g$. (At Greenwich)	= 1·50773.
6.	A carat	= $3\frac{1}{6}$ grains.
7.	Standard Silver	= 37 parts Silver + 3 parts Copper.
8.	Standard Gold.	= 22 parts Gold + 2 parts Copper.
9.	Mint price of Standard Silver	= 5*s*. 6*d*. per ounce Troy.
10.	Mint price of Standard Gold.	= £3 17*s*. $10\frac{1}{2}$*d*. ,, ,,
11.	30 Cubic inches of Gunpowder	= 1 lb.
12.	An iron ball 4 inches diameter	= 9 lbs.
13.	A leaden bullet 1 inch diameter	= $\frac{3}{14}$ lb.
14.	Seconds' Pendulum, Length (l) at Greenwich .	= 39·1393 inches.
15.	$\log_{10} l$	= 1·59261.
16.	Cubic in. distilled water (Bar. 30 in. Ther. 62°)	= 252·458 grains.
17.	Cubic foot ,, ,, ,, ,,	= 908·8488 ounces Troy.
18.	,, ,, ,, ,, ,,	= 997·136969 ,, Avoir.
19.	Barometer, Standard Height at 32° F. . .	= 30 inches.
20.	Height of a Column of Water supported by Atmosphere	= 34 feet.
21.	Mean Pressure of the Atmosphere	= 15 lbs. per square inch nearly.
22.	Velocity of Light	= 192000 miles per second.
23.	Earth's Equatorial Radius	= 20923713 feet = 3962·82 miles.
24.	,, Polar ,, ...	= 20853810 ,, = 3949·59 ,,
25.	Mean distance of the Earth from the Sun . .	= 95000000 miles.
26.	Distance of the Earth from the Moon . . .	= 59·964 × Earth's radius.
27.	Length of 1° in Latitude 45°	= 364543·5 feet.
28.	Geographical Mile in Latitude $50\frac{1}{2}$°	= 1·152 statute miles.
29.	Mean Solar Day	= 24 h. 3 m. 56·554 s. sid. time.
30.	Atmosphere. *Assumed* Height.	= 27690 feet.
31.	Expansion of Mercury for 1° Fahr.	= 0·00010415.
32.	Vol. of Steam generated by Cubic foot of water	= 883 Cubic feet.
33.	Pressure of ditto per square inch	= 30 lbs.

THE END.

R. CLAY, SON, AND TAYLOR, PRINTERS, LONDON.

186, Fleet Street,
March, 1862.

MESSRS. BELL AND DALDY'S

NEW AND STANDARD PUBLICATIONS.

New Books.

SPENSER'S Complete Works: with Life, Notes, and a Glossary, by John Payne Collier, Esq., F.S.A. 8vo. 5 vols. 3*l.* 15*s.* Antique calf, 6*l.* 6*s.* [*Ready.*

Aldine Poets.—Thomson's Poems, with Memoir by Sir H. Nicolas, and additional Poems; the whole very carefully revised, and the Memoir annotated by Peter Cunningham, Esq., F.S.A. 2 vols. 10*s.* Morocco, or antique morocco, 21*s.* [*Ready.*

Aldine Poets.—Thomson's Seasons, and Castle of Indolence, with Memoir, 6*s.* Morocco, or antique morocco, 11*s.* 6*d.* [*Ready.*

Poems from the German. By Richard Garnett, Author of "Io in Egypt, and other Poems." Fcap. 8vo. 3*s.* 6*d.* [*Ready.*

Saul, a Dramatic Poem; Elizabeth, an Historical Ode; and other Poems. By William Fulford, M.A. Fcap. 8vo. 5*s.* [*Ready.*

Lays and Poems on Italy. By F. A. Mackay. [*In the press.*

The Book of Common Prayer. Ornamented with Head-pieces and Initial Letters specially designed for this edition. Printed in red and black at the Cambridge University Press. 24mo. Best morocco, 10*s*. 6*d*. Also in ornamental bindings, at various prices. [*Ready*.

Also a large paper Edition, crown 8vo. Best morocco, 18*s*. Also in ornamental bindings, at various prices. [*Ready*.

A Commentary on the Gospels for the Sundays and other Holy Days of the Christian Year. By the Rev. W. Denton, A.M., Worcester College, Oxford, and Incumbent of St. Bartholomew's, Cripplegate. Vol. II. Easter to the Sixteenth Sunday after Trinity. 8vo. 14*s*. [*Ready*.

Parish Sermons. By the Rev. M. F. Sadler, M. A., Vicar of Bridgwater. Author of the "Sacrament of Responsibility," and "The Second Adam and the New Birth." Vol. II Trinity. Fcap. 8vo. [*In the Press*.

The Second Adam, and the New Birth; or, the Doctrine of Baptism as contained in Holy Scripture. By the Rev. M. F. Sadler, M. A., Vicar of Bridgwater, Author of "The Sacrament of Responsibility." *Third Edition, greatly enlarged*. Fcap. 8vo. 4*s*. 6*d*. [*Ready*.

Sermons on Popular Subjects, preached in the Collegiate Church, Wolverhampton. By the Rev. Julius Lloyd, M.A. Small 8vo. [*Immediately*.

Readings for the Sundays from Advent to Trinity. By a Clergyman's Daughter. Fcap. 8vo. Stiff covers, 1*s*. [*Ready*.

Reasons of Faith; or the Christian Argument gradually developed and explained. By the Rev. J. S. Drew, M.A. [*In the Press*.

Popular Illustrations of some Remarkable Events recorded in the Old Testament. By the Rev. J. F. Dawson, LL.B., Rector of Toynton. Post 8vo. 8*s*. 6*d*. [*Ready*.

The Book of Psalms; a New Translation, with Introductions and Notes, Critical and Explanatory. By the Rev. J. J. Stewart Perowne, B.D., Fellow of C. C. College, Cambridge, and Examining Chaplain to the Lord Bishop of Norwich. 8vo. [*In the Press*.

Scudamore Organs, or Practical Hints respecting Organs for Village Churches and Small Chancels, on improved Principles. By the Rev. John Baron, M.A., Rector of Upton Scudamore, Wilts. With Designs by G. E. Street, F.S.A. *Second Edition, revised*. [*In the Press*.

A History of Ecclesfield. By the Rev. J. Eastwood. 8vo. [*In the Press*.

The Cotton, Flax, and other Chief Fibre-yielding Plants of India; with a coloured Map of the Country, several original Illustrations of the Native Fibrous Plants, and many important Statistical Tables. By J. Forbes Watson, A.M., M.D., Reporter to the Indian Government on the Products of India. Royal 8vo. [*Immediately*.

A Handy Book of the Chemistry of Soils. By John Scoffern, M.B. Crown 8vo. [*Shortly*.

Notes and Queries.—General Index to the Second Series. Fcap. 4to. 5*s*. [*Immediately*.

Childhood and Youth. By Count Nicola Tolstoi. Translated from the Russian by Malwida von Meysenbug. Post 8vo. 8s. 6d. [*Ready.*

Gifts and Graces. A new Tale, by the Author of "The Rose and the Lotus." Post 8vo. 7s. 6d. [*Ready.*

Baronscliffe; or, the Deed of other Days. By Mrs. P. M. Latham, Author of "The Wayfarers." Crown 8vo. 6s. [*Ready.*

The Old Folks from Home; or, a Holiday in Ireland in 1861. By Mrs. Gatty. Post 8vo. 7s. 6d. [*Ready.*

Domestic Life in Palestine. By M. E. Rogers. Post 8vo. 10s. 6d. [*Ready.*

The Lights of the Will o' the Wisp. Translated by Lady Maxwell Wallace. With a coloured Frontispiece. Imperial 16mo. Cloth, gilt edges, 5s. [*Ready.*

The Children's Picture Book of Useful Knowledge. With 130 Illustrations. Uniform with "The Children's Bible Picture Book." Super royal 16mo. Cloth, gilt edges, 5s. [*Ready.*

The Schole Master. By Roger Ascham. Edited by the Rev. J. E. B. Mayor, M. A. Fcap. 8vo. [*Shortly.*

Under Government: an Official Key to the Civil Service, and Guide for Candidates seeking Appointments under the Crown. By Joseph Charles Parkinson, Inland Revenue, Somerset House. *New Edition.* [*Preparing.*

The Elements of the English Language, for Schools and Colleges. By Ernest Adams, Ph. D., University College School. *New Edition, enlarged and improved.* [*Nearly ready.*

A Latin Grammar. By T. Hewitt Key, M. A., F.R.S., Professor of Comparative Grammar, and Head Master of the Junior School, in University College. *Third Edition, revised.* Post 8vo. 8s. [*Ready.*

Latin Prose Lessons. By the Rev. A. Church, M.A., one of the Masters of Merchant Taylors' School. Fcap. 8vo. 2s. 6d. [*Ready.*

Gasc's Le Petit Compagnon: a French Talk-book for Little Children. With numerous wood-cuts. 16mo. 3s. 6d. [*Immediately.*

Gasc's Select French Poetry for the Young. With English Notes, and preceded by a few Plain Rules of French Prosody. Fcap. 8vo. 2s. [*Ready.*

A Key to M. Gasc's First and Second French Books. Fcap. 8vo. 3s. 6d. [*Ready.*

A Compendium of Facts and Formulæ in Pure and Mixed Mathematics. For the use of Mathematical Students. By G. R. Smalley, B.A., F.R.A.S. Fcap. 8vo. [*Immediately.*

The Choephorae of Aeschylus and its Scholia. Revised and interpreted by J. F. Davies, Esq., B. A., Trinity College, Dublin. 8vo. [*In the Press.*

Bell and Daldy's POCKET VOLUMES. A Series of Select Works of Favourite Authors, adapted for general reading, moderate in price, compact and elegant in form, and executed in a style fitting them to be permanently preserved. Imperial 32mo.

Now Ready.

The Midshipman.—Autobiographical Sketches of his own Early Career, by Capt. Basil Hall, R.N., F.R.S. From his "Fragments of Voyages and Travels." 3*s.*
Southey's Life of Nelson. 2*s.* 6*d.*
George Herbert's Poems. 2*s.*
George Herbert's Works. 3*s.*
Longfellow's Poems. 2*s.* 6*d.*
Lamb's Tales from Shakspeare. 2*s.* 6*d.*
Milton's Paradise Lost. 2*s.* 6*d.*
Milton's Paradise Regained and other Poems. 2*s.* 6*d.*

Preparing.

The Robin Hood Ballads.
Sea Songs and Ballads, by Charles Dibdin, and others.
The Lieutenant and Commander.—Autobiographical Sketches of his own Early Career, by Capt. Basil Hall, R. N., F. R. S. From his "Fragments of Voyages and Travels."
Walton's Lives of Donne, Wotton, Hooker, &c.
Walton's Complete Angler.

In cloth, top edge gilt, at 6*d.* per volume extra; in half morocco, Roxburgh style, at 1*s.* extra; in antique or best plain morocco (Hayday) at 4*s.* extra.

Other Works are in preparation.

The Library of English Worthies.

A Series of reprints of the best Authors carefully edited and collated with the Early Copies, and handsomely printed by Whittingham in Octavo.

GOWER'S Confessio Amantis, with Life by Dr. Pauli, and a Glossary. 3 vols. 2*l.* 2*s.* Antique calf, 3*l.* 6*s.* Only a limited number of Copies printed.

This important work is so scarce that it can seldom be met with even in large libraries. It is wanting in nearly every collection of English Poetry.

Spenser's Complete Works; with Life, Notes, and Glossary, by John Payne Collier, Esq., F.S.A. 5 vols. 3*l.* 15*s.* Antique calf, 6*l.* 6*s.*

Bishop Butler's Analogy of Religion; with Analytical Index, by the Rev. Edward Steere, LL.D. 12*s.* Antique calf, 1*l.* 1*s.*

"The present edition has been furnished with an Index of the Texts of Scripture quoted, and an Index of Words and Things considerably fuller than any hitherto published."—*Editor's Preface.*

Bishop Jeremy Taylor's Rule and Exercises of Holy Living and Dying. 2 vols. 1*l.* 1*s.* Morocco, antique calf or morocco, 2*l.* 2*s.*

Library of English Worthies continued.

Herbert's Poems and Remains; with S. T. Coleridge's Notes, and Life by Izaak Walton. Revised, with additional Notes, by Mr. J. Yeowell. 2 vols. 1*l.* 1*s.* Morocco, antique calf or morocco, 2*l.* 2*s.*

Uniform with the above.

The Physical Theory of Another Life. By Isaac Taylor, Esq., Author of "Logic in Theology," "Ultimate Civilization, &c." *New Edition.* 10*s.* 6*d.* Antique calf, 21*s.*

The Aldine Edition of the British Poets.

The Publishers have been induced, by the scarcity and increasing value of this admired Series of the Poets, to prepare a New Edition, very carefully corrected, and improved by such additions as recent literary research has placed within their reach.

The general principle of Editing which has been adopted is *to give the entire Poems of each author in strict conformity with the Edition which received his final revision, to prefix a Memoir,* and *to add such notes as may be necessary to elucidate the sense of obsolete words or explain obscure allusions.* Each author will be placed in the hands of a competent editor specially acquainted with the literature and bibliography of the period.

Externally this new edition will resemble the former, but with some improvements. It will be elegantly printed by Whittingham, on toned paper manufactured expressly for it; and a highly-finished portrait of each author will be given.

The *Aldine Edition of the British Poets* has hitherto been the favourite Series with the admirers of choice books, and every effort will be made to increase its claims as a comprehensive and faithful mirror of the poetic genius of the nation.

AKENSIDE'S Poetical Works, with Memoir by the Rev. A. Dyce, and additional Letters, carefully revised. 5*s.* Morocco, or antique morocco, 10*s.* 6*d.*

Collins's Poems, with Memoir and Notes by W. Moy Thomas, Esq. 3*s.* 6*d.* Morocco, or antique morocco, 8*s.* 6*d.*

Gray's Poetical Works, with Notes and Memoir by the Rev. John Mitford. 5*s.* Morocco, or antique morocco, 10*s.* 6*d.*

Kirke White's Poems, with Memoir by Sir H. Nicolas, and additional notes. Carefully revised. 5*s.* Morocco, or antique morocco, 10*s.* 6*d.*

Shakespeare's Poems, with Memoir by the Rev. A. Dyce. 5*s.* Morocco, or antique morocco, 10*s.* 6*d.*

Young's Poems, with Memoir by the Rev. John Mitford, and additional Poems. 2 vols. 10*s.* Morocco, or antique morocco, 1*l.* 1*s.*

Aldine Poets continued.

Thomson's Poems, with Memoir by Sir H. Nicolas, and additional Poems; the whole very carefully revised, and the Memoir annotated by Peter Cunningham, Esq., F.S.A. 2 vols. 10*s*. Morocco, or antique morocco, 1*l*. 1*s*.

Thomson's Seasons, and Castle of Indolence, with Memoir. 6*s*. Morocco, or antique morocco, 11*s*. 6*d*.

Dryden's Poetical Works, with Memoir by the Rev. R. Hooper, F.S.A. Carefully revised. [*In the Press.*

Cowper's Poetical Works, including his Translations. Edited, with Memoir, by John Bruce, Esq., F.S.A. [*Preparing.*

Parnell's Poems, with Memoir, edited by Bolton Corney, Esq., M.R.S.L. [*Preparing.*

Pope's Poetical Works, with Memoir. Edited by W. J. Thoms, Esq., F.S.A. [*Preparing.*

MR. S. W. Singer's New Edition of Shakespeare's Dra- matic Works. The Text carefully revised, with Notes. The Life of the Poet and a Critical Essay on each Play by W. W. Lloyd, Esq. 10 vols. 6*s*. each. Calf, 5*l*. 5*s*. Morocco, 6*l*. 6*s*.

Large Paper Edition, crown 8vo., 4*l*. 10*s*. Calf, 6*l*. 16*s*. 6*d*. Morocco, 8*l*. 8*s*.

"Mr. Singer has produced a text, the accuracy of which cannot be surpassed in the present state of antiquarian and philological knowledge."—*Daily News*.

DR. RICHARDSON'S New Dictionary of the English Language. Combining Explanation with Etymology, and copiously illustrated by Quotations from the best authorities. *New Edition*, with a Supplement containing additional Words and further Illustrations. In Two Vols. 4to. 4*l*. 14*s*. 6*d*. Half bound in russia, 5*l*. 15*s*. 6*d*. Russia, 6*l*. 12*s*.

The WORDS—with those of the same Family—are traced to their Origin.

The EXPLANATIONS are deduced from the Primitive Meaning through the various Usages.

The QUOTATIONS are arranged Chronologically, from the Earliest Period to the Present Time.

*** The Supplement separately, 4to. 12*s*.

AN 8VO. EDITION, without the Quotations, 15*s*. Half-russia, 20*s*. Russia, 24*s*.

"It is an admirable addition to our Lexicography, supplying a great desideratum, as exhibiting the biography of each word—its birth, parentage and education, the changes that have befallen it, the company it has kept, and the connexions it has formed—by rich series of quotations, all in chronological order. This is such a Dictionary as perhaps no other language could ever boast."—*Quarterly Review*.

Dr. Richardson on the Study of Language: an Exposition of Horne Tooke's Diversions of Purley. Fcap. 8vo. 4*s*. 6*d*.

Uniform with the Aldine Edition of the Poets.

The Works of Gray, edited by the Rev. John Mitford. With his Correspondence with Mr. Chute and others, Journal kept at Rome, Criticism on the Sculptures, &c. *New Edition.* 5 vols. 1*l.* 5*s.*

The Temple and other Poems. By George Herbert, with Coleridge's Notes. *New Edition.* 5*s.* Morocco, antique calf or morocco, 10*s.* 6*d.*

Vaughan's Sacred Poems and Pious Ejaculations, with Memoir by the Rev. H. F. Lyte. *New Edition.* 5*s.* Antique calf or morocco, 10*s.* 6*d.* *Large Paper,* 7*s.* 6*d.* Antique calf, 14*s.* Antique morocco, 15*s.*

"Preserving all the piety of George Herbert, they have less of his quaint and fantastic turns, with a much larger infusion of poetic feeling and expression."—*Lyte.*

Bishop Jeremy Taylor's Rule and Exercises of Holy Living and Holy Dying. 2 vols. 2*s.* 6*d.* each. Morocco, antique calf or morocco, 7*s.* 6*d.* each. In one volume, 5*s.* Morocco, antique calf or morocco, 10*s.* 6*d.*

Bishop Butler's Analogy of Religion; with Analytical Introduction and copious Index, by the Rev. Dr. Steere. 6*s.* Antique calf, 11*s.* 6*d.*

Bishop Butler's Sermons and Remains; with Memoir, by the Rev. E. Steere, LL.D. 6*s.*

*** This volume contains some additional remains, which are copyright, and render it the most complete edition extant.

Bishop Butler's Complete Works; with Memoir by the Rev. Dr. Steere. 2 vols. 12*s.*

Bacon's Advancement of Learning. Edited, with short Notes, by the Rev. G. W. Kitchin, M.A., Christ Church, Oxford. 6*s.*; antique calf, 11*s.* 6*d.*

Bacon's Essays; or, Counsels Civil and Moral, with the Wisdom of the Ancients. With References and Notes by S. W. Singer, F.S.A. 5*s.* Morocco, or antique calf, 10*s.* 6*d.*

Bacon's Novum Organum. Newly translated, with short Notes, by the Rev. Andrew Johnson, M.A. 6*s.* Antique calf, 11*s.* 6*d.*

Locke on the Conduct of the Human Understanding; edited by Bolton Corney, Esq., M. R. S. L. 3*s.* 6*d.* Antique calf, 8*s.* 6*d.*

"I cannot think any parent or instructor justified in neglecting to put this little treatise into the hands of a boy about the time when the reasoning faculties become developed."—*Hallam.*

Ultimate Civilization. By Isaac Taylor, Esq. 6*s.*

Logic in Theology, and other Essays. By Isaac Taylor, Esq. 6*s.*

The Physical Theory of Another Life. By Isaac Taylor, Esq., Author of the "Natural History of Enthusiasm," "Restoration of Belief," &c. *New Edition.* 6*s.* Antique calf, 11*s.* 6*d.*

DOMESTIC Life in Palestine. By M. E. Rogers. Post 8vo. 10*s*. 6*d*.

By-Roads and Battle Fields in Picardy: with Incidents and Gatherings by the Way between Ambleteuse and Ham; including Agincourt and Crécy. By G. M. Musgrave, M.A., Author of "A Pilgrimage into Dauphiné," &c. Illustrated. Super-royal 8vo. 16*s*.

The Boat and the Caravan. A Family Tour through Egypt and Syria. *New and cheaper Edition.* Fcap. 8vo. 5*s*. 6*d*.

Fragments of Voyages and Travels. By Captain Basil Hall, R.N., F.R.S. 1st, 2nd, and 3rd Series in 1 vol. complete. *New Edition.* Royal 8vo. 10*s*. 6*d*.

The Gem of Thorney Island; or, The Historical Associations of Westminster Abbey. By the Rev. J. Ridgway, M.A. Crown 8vo. 7*s*. 6*d*.

The Life and Times of Aonio Paleario; or, a History of the Italian Reformers in the Sixteenth Century. Illustrated by Original Letters and unedited Documents. By M. Young. 2 vols. 8vo. 1*l*. 12*s*.

Claude de Vesci; or, the Lost Inheritance. 2 vols. Fcap. 8vo. 9*s*.

Maud Bingley. By Frederica Graham. 2 vols. Fcap. 8vo. 12*s*.

The Wayfarers: or, Toil and Rest. By Mrs. Latham. Fcap. 5*s*.

The Manse of Mastland. Sketches: Serious and Humorous, in the Life of a Village Pastor in the Netherlands. Translated from the Dutch by Thomas Keightley, M.A. Post 8vo. 9*s*.

The Home Life of English Ladies in the Seventeenth Century. By the Author of "Magdalen Stafford." *Second Edition, enlarged.* Fcap. 8vo. 6*s*. Calf, 9*s*. 6*d*.

The Romance and its Hero. By the Author of "Magdalen Stafford." 2 vols. Fcap. 8vo. 12*s*.

Magdalen Stafford. A Tale. Fcap. 8vo. 5*s*.

BY THE LATE MRS. WOODROOFFE.

COTTAGE Dialogues. *New Edition.* 12mo. 4*s*. 6*d*.

Shades of Character; or, the Infant Pilgrim. *7th Edition.* 2 vols. 12mo. 12*s*.

Michael Kemp, the Happy Farmer's Lad. *8th Edition.* 12mo. 4*s*.

A Sequel to Michael Kemp. *New Edition.* 12mo. 6*s*. 6*d*.

MRS. ALFRED GATTY'S POPULAR WORKS.

"We should not be doing justice to the highest class of juvenile fiction, were we to omit, as particularly worthy of attention at this season, the whole series of Mrs. Gatty's admirable books. They are quite *sui generis*, and deserve the widest possible circulation."—*Literary Churchman.*

PARABLES from Nature; with Notes on the Natural History. Illustrated by W. Holman Hunt, Otto Speckter, C. W. Cope, R. A., E. Warren, W. Millais, G. Thomas, and H. Calderon. 8vo. Ornamental cloth, 10*s.* 6*d.* Antique morocco elegant, 1*l.* 1*s.*

Parables from Nature. 16mo. with Illustrations. *Tenth Edition.* 3*s.* 6*d.* Separately: First Series, 1*s.* 6*d.*; Second Series, 2*s.*

Red Snow, and other Parables from Nature. Third Series, with Illustrations. *Second Edition.* 16mo. 2*s.*

Worlds not Realized. 16mo. *Third Edition.* 2*s.*

Proverbs Illustrated. 16mo. with Illustrations. *3rd Edition.* 2*s.*

*** *These little works have been found useful for Sunday reading in the family circle, and instructive and interesting to school children.*

Aunt Judy's Tales. Illustrated by Clara S. Lane. Fcap. 8vo. *Third Edition.* 3*s.* 6*d.*

The Human Face Divine, and other Tales. With Illustrations by C. S. Lane. Fcap. 8vo. 3*s.* 6*d.*

The Fairy Godmothers and other Tales. *Third Edition.* Fcap. 8vo. with Frontispiece. 2*s.* 6*d.*

Legendary Tales. With Illustrations by Phiz. Fcap. 8vo. 5*s.*

The Poor Incumbent. Fcap. 8vo. Sewed, 1*s.* Cloth, 1*s.* 6*d.*

The Old Folks from Home; or, a Holiday in Ireland in 1861. Post 8vo. 7*s.* 6*d.*

THE Life and Adventures of Robinson Crusoe. By Daniel Defoe. With 100 Illustrations by E. H. Wehnert. Uniform with "Andersen'sTales." Small 8vo. Cloth, gilt edges, 7*s.* 6*d.*

Andersen's Tales for Children. Translated by A. Wehnert. With 105 Illustrations by E. H. Wehnert, W. Thomas, and others. Small 8vo. Cloth, gilt edges, 7*s.* 6*d.*

Among the Tartar Tents; or, the Lost Fathers. A Tale By Anne Bowman, Author of "Esperanza," "The Boy Voyagers," &c. With Illustrations. Crown 8vo. 5*s.*

Nursery Carols. Illustrated with 120 Pictures. By Ludwig Ricther and Oscar Pletsch. Imperial 16mo. Ornamental Binding. 3*s.* 6*d.* coloured, 6*s.*

Little Maggie and her Brother. By Mrs. G. Hooper, Author of "Recollections of Mrs. Anderson's School," "Arbell," &c. With a Frontispiece. Fcap. 8vo. 2*s*. 6*d*.

Guessing Stories; or, the Surprising Adventures of the Man with the Extra Pair of Eyes. A Book for Young People. By a Country Parson. Imperial 16mo. Cloth, gilt edges, 3*s*.

Cavaliers and Round Heads. By J. G. Edgar, Author of "Sea Kings and Naval Heroes." Illustrated by Amy Butts. Fcap. 8vo. 5*s*.

Sea-Kings and Naval Heroes. A Book for Boys. By J. G. Edgar. With Illustrations by C. K. Johnson and C. Keene. Fcap. 8vo. 5*s*.

The Children's Picture Book of Good and Great Men. Written expressly for Young People, and Illustrated with Fifty large Engravings. Super royal 16mo. Cloth, gilt edges, 5*s*. With Coloured Illustrations, 9*s*.

The Children's Picture Book of English History. Written expressly for Young People, and Illustrated with Sixty large Engravings. Super royal 16mo. Cloth, gilt edges, 5s. With Coloured Illustrations, 9*s*.

The Children's Bible Picture Book. Written expressly for Young People, and Illustrated with Eighty large Engravings. *Third Edition.* Super royal 16mo. Cloth, gilt edges, 5*s*. With Coloured Illustrations, 9*s*.

The Children's Picture Book of Scripture Parables and Bible Miracles, in 1 vol. Cloth, gilt edges, 5*s*. Coloured, 7*s*. 6*d*.

The Children's Picture Book of Scripture Parables. By the Rev. J. Erskine Clarke. With Sixteen large Illustrations. Super royal 16mo. Cloth, red edges, 2*s*. 6*d*. Coloured, with gilt edges, 3*s*. 6*d*.

The Children's Picture Book of Bible Miracles. By the Rev. J. Erskine Clarke, M.A. With Sixteen large Illustrations. Super royal 16mo. Cloth, red edges, 2*s*. 6*d*. Coloured, with gilt edges, 3*s*. 6*d*.

The Children's Bunyan's Pilgrim's Progress. With Sixteen large Illustrations. *New Edition.* Super royal 16mo. Cloth, red edges, 2*s*. 6*d*. Coloured, with gilt edges, 3*s*. 6*d*.

The Children's Picture Book of the Life of Joseph, written in Simple Language. With Sixteen large Illustrations. Super royal 16mo. Cloth, red edges, 2*s*. 6*d*. Coloured, with gilt edges, 3*s*. 6*d*.

The Life of Christopher Columbus, in Short Words. By Sarah Crompton. Crown 8vo. 2*s*. 6*d*. Also an Edition for Schools, 1*s*.

The Life of Martin Luther, in Short Words. By the same Author. Crown 8vo. 1*s*. 6*d*. Stiff cover, 1*s*.

Redfield; or, a Visit to the Country. A Story for Children. With Illustrations by Absolon. Super royal 16mo. 2*s*. 6*d*. Coloured, 3*s*. 6*d*.

Nursery Tales. By Mrs. Motherly. With Illustrations by C. S. Lane. Imperial 16mo. 2*s*. 6*d*. Coloured, gilt edges, 3*s*. 6*d*.

Nursery Poetry. By Mrs. Motherly. With Eight Illustrations by C. S. Lane. Imperial 16mo. 2*s*. 6*d*. Coloured, gilt edges, 3*s*. 6*d*.

Poetry for Play-Hours. By Gerda Fay. With Eight large Illustrations. Imperial 16mo. 3*s.* 6*d.* Coloured, gilt edges, 4*s.* 6*d.*

Very Little Tales for Very Little Children. In single Syllables of *Four* and *Five* letters. *New Edition.* Illustrated. 2 vols. 16mo. 1*s.* 6*d.* each, or in 1 vol. 3*s.*

Progressive Tales for Little Children. In words of *One* and *Two* Syllables. Forming the sequel to "Very Little Tales." *New Edition.* Illustrated. 2 vols. 16mo. 1*s.* 6*d.* each, or in 1 vol. 3*s.*

The White Lady and Undine, translated from the German by the Hon. C. L. Lyttelton. With numerous Illustrations. Fcap. 8vo. 5*s.* Or, separately, 2*s.* 6*d.* each.

The Lights of the Will o' the Wisp. Translated by Lady Maxwell Wallace. With a coloured Frontispiece. Imperial 16mo. Cloth, gilt edges, 5*s.* [*Ready.*

Voices from the Greenwood. Adapted from the Original. By Lady Maxwell Wallace. With Illustrations. Imperial 16mo. 2*s.* 6*d.*

Princess Ilse: a Legend, translated from the German. By Lady Maxwell Wallace. With Illustrations. Imperial 16mo. 2*s.* 6*d.*

A Poetry Book for Children. Illustrated with Thirty-seven highly-finished Engravings, by C. W. Cope, R. A., Helmsley, Palmer, Skill, Thomas, and H. Weir. *New Edition.* Crown 8vo. 2*s.* 6*d.*

CLARK'S Introduction to Heraldry.—Containing Rules for Blazoning and Marshalling Coats of Armour—Dictionary of Terms—Orders of Knighthood explained—Degrees of the Nobility and Gentry—Tables of Precedency; 48 Engravings, including upwards of 1,000 Examples, and the Arms of numerous Families. *Sixteenth Edition improved.* Small 8vo. 7*s.* 6*d.* Coloured, 18*s.*

Book of Family Crests and Mottoes, with *Four Thousand Engravings* of the Crests of the Peers, Baronets, and Gentry of England and Wales, and Scotland and Ireland. A Dictionary of Mottos, &c. *Ninth Edition, enlarged.* 2 vols. small 8vo. 1*l.* 1*s.*

"Perhaps the best recommendation to its utility and correctness (in the main) is, that it has been used as a work of reference in the Heralds College. No wonder it sells."—*Spectator.*

Book of Mottoes, used by the Nobility, Gentry, &c. with Translations, &c. *New Edition, enlarged.* Small 8vo. cloth gilt, 2*s.* 6*d.*

A Handbook of Mottoes borne by the Nobility, Gentry, Cities, Public Companies, &c. Translated and Illustrated, with Notes and Quotations, by C. N. Elvin, M.A. Small 8vo. 6*s.*

Gothic Ornaments; being a Series of Examples of enriched Details and Accessories of the Architecture of Great Britain. Drawn from existing Authorities. By J. K. Colling, Architect. Royal 4to. Vol. I. 3*l.* 13*s.* 6*d.* Vol. II. 3*l.* 16*s.* 6*d.*

Details of Gothic Architecture, Measured and Drawn from existing Examples. By J. K. Colling, Architect. Royal 4to. 2 vols. *5l. 5s.*

The Architectural History of Chichester Cathedral, with an Introductory Essay on the Fall of the Tower and Spire. By the Rev. R. Willis, M.A., F.R.S., &c., Jacksonian Professor in the University of Cambridge.—Of Boxgrove Priory, by the Rev. J. L. Petit, M.A., F.S.A. —And of Shoreham Collegiate Church, together with the Collective Architectural History of the foregoing buildings, as indicated by their mouldings, by Edmund Sharpe, M.A., F.R.I.B.A. Illustrated by one hundred Plates, Diagrams, Plans and Woodcuts. Super-royal 4to. *1l. 10s.*

Architectural Studies in France. By the Rev. J. L. Petit, M.A., F.S.A. With Illustrations from Drawings by the Author and P. H. Delamotte. Imp. 8vo. *2l. 2s.*

Remarks on Church Architecture. With Illustrations. By the Rev. J. L. Petit, M.A. 2 vols. 8vo. *1l. 1s.*

Lectures on Church Building: with some Practical Remarks on Bells and Clocks. By E. B. Denison, M.A. *Second Edition.* Rewritten and enlarged; with Illustrations. Crown 8vo. *7s. 6d.*

A Few Notes on the Temple Organ. By Edmund Macrory, M.A. *Second Edition.* Super-royal 16mo. Half morocco, Roxburgh, *3s. 6d.*

Scudamore Organs, or Practical Hints respecting Organs for Village Churches and small Chancels, on improved principles. By the Rev. John Baron, M.A., Rector of Upton Scudamore, Wilts. With Designs by George Edmund Street, F.S.A. 8vo. *5s.*

Memoirs of Musick. By the Hon. Roger North, Attorney-General to James II. Now first printed from the original MS., and edited, with copious Notes, by Dr. E. F. Rimbault. Fcap. 4to. half morocco, *1l. 10s.*

The Bell; its Origin, History, and Uses. By Rev. A. Gatty. *3s.*

Practical Remarks on Belfries and Ringers. By the Rev. H. T. Ellacombe, M.A., F.A.S., Rector of Clyst St. George, Devonshire. *Second Edition*, with an Appendix on Chiming. Illustrated. 8vo. *3s.*

Proceedings of the Archæological Institute at Newcastle, in 1853. With Numerous Engravings. 2 vols. 8vo. *2l. 2s.*

A Handbook for Visitors to Cambridge. By Norris Deck. Illustrated by 8 Steel Engravings, 97 Woodcuts, and a Map. Crown 8vo. *5s.*

Canterbury in the Olden Time: from the Municipal Archives and other Sources. By John Brent, F.S.A. With Illustrations. *5s.*

Whirlwinds and Dust-Storms of India. With numerous Illustrations drawn from Nature, bound separately; and an Addendum on Sanitary Measures required for European Soldiers in India. By P. F. H. Baddeley, Surgeon, Bengal Army, Retired List. Large 8vo. With Illustrations, *8s. 6d.*; without Illustrations, *3s.*

Two Transparent Wind Cards in Horn, adapted to the Northern and Southern Hemispheres, for the use of Sailors. *2s.*

Tables of Interest, enlarged and Improved; calculated at Five per Cent.; Showing at one view the Interest of any Sum, from £1 to £365: they are also carried on by hundreds to £1,000, and by thousands to £10,000, from one day to 365 days. To which are added, Tables of Interest, from one to 12 months, and from two to 13 years. Also Tables for calculating Commission on Sales of Goods or Banking Accounts, from ⅛ to 5 per Cent., with several useful additions, among which are Tables for calculating Interest on large sums for 1 day, at the several rates of 4 and 5 per Cent. to £100,000,000. By Joseph King, of Liverpool. *24th Edition.* With a Table showing the number of days from any one day to any other day in the Year. 8vo. 1*l.* 1*s.*

WEBSTER'S Complete Dictionary of the English Language. *New Edition*, revised and greatly enlarged, by CHAUNCEY A. GOODRICH, Professor in Yale College. 4to. (1624 pp.) 1*l.* 11*s.* 6*d.*; half calf, 2*l.*; calf, or half russia, 2*l.* 2*s.*; russia, 2*l.* 10*s.*

Though the circulation of Dr. Webster's celebrated Dictionary, in its various forms, in the United States, in England, and in every country where the English Language is spoken, may be counted by hundreds of thousands, it is believed that there are many persons to whom the book is yet unknown, and who, if seeking for a Dictionary which should supply all reasonable wants, would be at a loss to select one from the numerous competitors in the field.

In announcing this New Edition, the Proprietors desire to call attention to the features which distinguish it, and to put before those who are in want of such a book, the points in which it excels all other Dictionaries, and which render it the best that has as yet been issued for the practical purposes of daily use:—

1. Accuracy of Definition. 2. Pronunciation intelligibly marked. 3. Completeness. 4. Etymology. 5. Obsolete Words. 6. Uniformity in the Mode of Spelling. 7. Quotations. 8. Cheapness.

With the determination that the superiority of the work shall be fully maintained, and that it shall keep pace with the requirements of the age and the universal increase of education, the Proprietors have added to this New Edition, under the editorship of Professor Goodrich,—

A Table of Synonyms. An Appendix of New Words. Table of Quotations, Words, Phrases, &c.

LEGENDS and Lyrics, by Adelaide Anne Procter. *6th Edition.* Fcap. 5*s.* Antique or best plain morocco, 10*s.* 6*d.*

——— *Second Series. Second Edition.* Fcap. 8vo. 5*s.*; antique or best plain morocco, 10*s.* 6*d.*

The Legend of the Golden Prayers, and other Poems. By C. F. Alexander, Author of "Moral Songs," &c. Fcap. 8vo. 5*s.*; antique or best plain morocco, 10*s.* 6*d.*

Verses for Holy Seasons. By the Same Author. Edited by the Very Rev. W. F. Hook, D.D. *4th Edition.* Fcap. 3*s.* 6*d.*; morocco, antique calf or morocco, 8*s.* 6*d.*

Nightingale Valley: a Collection of the Choicest Lyrics and Short Poems in the English Language. Fcap. 8vo. 5*s.*; morocco, antique calf or morocco, 10*s.* 6*d.*

The Monks of Kilcrea, and other Poems. *Third Edition.* Post 8vo. 7*s*. 6*d*.

Christopheros, and other Poems. By the Ven. W. B. Mant, Archdeacon of Down. Crown 8vo. 6*s*.

Teuton. A Poem. By C. J. Riethmüller. Crown 8vo. 7*s*. 6*d*.

Dryope, and other Poems. By T. Ashe. Fcap. 8vo. 6*s*.

Poems. By Thomas Ashe. Fcap. 8vo. 5*s*.

Day and Night Songs and The Music Master, a Love Poem. By William Allingham. With Nine Illustrations. Fcap. 8vo. 6*s*. 6*d*.; morocco, 11*s*. 6*d*.

Wild Thyme. By E. M. Mitchell. Fcap. 8vo. 5*s*.

Lyrics and Idylls. By Gerda Fay. Fcap. 8vo. 4*s*.

Pansies. By Fanny Susan Wyvill. Fcap. 8vo. 5*s*.

Io in Egypt, and other Poems. By R. Garnett. Fcap. 8vo. 5*s*.

The Defence of Guenevere, and other Poems. By W. Morris. 5*s*.

Herbert's Priest to the Temple, or the Country Parson. Super-royal 32mo. cloth, red edges, 2*s*. Morocco, or antique calf, 5*s*.

David Mallet's Poems. With Notes and Illustrations by F. Dinsdale, LL.D., F.S.A. *New Edition.* Post 8vo. 10*s*. 6*d*.

Ballads and Songs of Yorkshire. Transcribed from private MSS., rare Broadsides, and scarce Publications; with Notes and a Glossary. By C. J. D. Ingledew, M.A., Ph.D., F.G.H.S., author of "The History of North Allerton." Fcap. 8vo. 6*s*.

Passion Week. By the Editor of "Christmas Tyde." With 16 Illustrations from Albert Durer. Imp. 16mo. 7*s*. 6*d*.; antique morocco, 14*s*.

Percy's Reliques of Ancient English Poetry. 3 vols. sm. 8vo. 15*s*. Half-bound, 18*s*. Antique calf, or morocco, 1*l*. 11*s*. 6*d*.

Ellis's Specimens of Early English Poetry. 3 vols. sm. 8vo. 15*s*. Half-bound, 18*s*. Antique calf, or morocco, 1*l*. 11*s*. 6*d*.

The Book of Ancient Ballad Poetry of Great Britain, Historical, Traditional and Romantic: with Modern Imitations, Translations, Notes and Glossary, &c. Edited by J. S. Moore. *New and Improved Edition*, 8vo. Half-bound, 14*s*. Antique morocco, 21*s*.

Shakespeare's Tempest. With Illustrations by Birket Foster, Gustave Doré, Frederick Skill, Alfred Slader, and Gustave Janet. Crown 4to. Ornamental cloth, 10*s*. 6*d*. Antique morocco elegant, 1*l*. 1*s*.

The Promises of Jesus Christ. Illuminated by Albert H. Warren, *Second Edition.* Ornamental cloth, 15*s*. Antique morocco elegant, 21*s*.

Christmas with the Poets: a Collection of English Poetry relating to the Festival of Christmas. Illustrated by Birket Foster, and with numerous initial letters and borders beautifully printed in gold and colours by Edmund Evans. *New and improved Edition.* Super royal 8vo. Ornamental binding, 21*s*. Antique morocco, 31*s*. 6*d*.

ATHENÆ Cantabrigienses. By C. H. Cooper, F.S.A., and Thompson Cooper. Volume I. 1500—1585. 8vo. 18*s*. Vol. II. 1586—1609. 8vo. 18*s*.

This work, in illustration of the biography of notable and eminent men who have been members of the University of Cambridge, comprehends notices of:—1. Authors. 2. Cardinals, archbishops, bishops, abbots, heads of religious houses and other church dignitaries. 3. Statesmen, diplomatists, military and naval commanders. 4. Judges and eminent practitioners of the civil or common law. 5. Sufferers for religious or political opinions. 6. Persons distinguished for success in tuition. 7. Eminent physicians and medical practitioners. 8. Artists, musicians, and heralds. 9. Heads of colleges, professors, and principal officers of the university. 10. Benefactors to the university and colleges, or to the public at large.

The Early and Middle Ages of England. By C. H. Pearson, M.A., Fellow of Oriel College, Oxford, and Professor of Modern History, King's College, London. 8vo. 12*s*.

History of England, from the Invasion of Julius Cæsar to the End of the Reign of George II., by Hume and Smollett. With the Continuation, to the Accession of Queen Victoria, by the Rev. T. S. Hughes, B.D. late Canon of Peterborough. *New Edition*, containing Historical Illustrations, Autographs, and Portraits, copious Notes, and the Author's last Corrections and Improvements. In 18 vols. crown 8vo. 4*s*. each.

Vols. I. to VI. (Hume's portion), 1*l*. 4*s*.
Vols. VII. to X. (Smollett's ditto), 16*s*.
Vols. XI. to XVIII. (Hughes's ditto), 1*l*. 12*s*.

History of England, from the Accession of George III. to the Accession of Queen Victoria. By the Rev. T. S. Hughes, B.D. *New Edition*, almost entirely re-written. In 7 vols. 8vo. 3*l*. 13*s*. 6*d*.

Choice Notes from "Notes and Queries," by the Editor. Fcap. 8vo. 5*s*. each.
VOL. I.—HISTORY. VOL. II.—FOLK LORE.

Master Wace's Chronicle of the Conquest of England. Translated from the Norman by Sir Alexander Malet, Bart., H.B.M. Plenipotentiary, Frankfort. With Photograph Illustrations of the Bayeaux Tapestry. Medium 4to. Half-morocco, Roxburgh, 2*l*. 2*s*.

The Prince Consort's Addresses on Different Public Occasions. Beautifully printed by Whittingham. 4to. 10*s*. 6*d*.

Life and Books; or, Records of Thought and Reading. By J. F. Boyes, M.A. Fcap. 8vo. 5*s*.; calf, 8*s*. 6*d*.

Life's Problems. *Second Edition*, revised and enlarged. Fcap. 5*s*.

Parliamentary Short-Hand (Official System). By Thompson Cooper. Fcap. 8vo. 2*s*. 6*d*.

This is the system *universally practised by the Government Official Reporters*. It has many advantages over the system ordinarily adopted, and has hitherto been inaccessible, except in a high-priced volume.

Geology in the Garden: or, The Fossils in the Flint Pebbles. With 106 Illustrations. By the Rev. Henry Eley, M.A. Fcap. 8vo. 6*s*.

The Pleasures of Literature. By R. Aris Willmott, Incumbent of Bear-Wood. *Fifth Edition*, enlarged. Fcap. 8vo. 5*s*. Morocco, 10*s*. 6*d*.

A Popular Sketch of the Origin and Progress of the English Language. By the Rev. W. W. Skeat, M.A. 12mo. 1*s*. 6*d*.

Hints to Maid Servants in Small Households, on Manners, Dress, and Duties. By Mrs. Motherly. Fcap. 8vo. 1*s*. 6*d*.

A Wife's Home Duties; containing Hints to inexperienced Housekeepers. Fcap. 8vo. 2*s*. 6*d*.

Halcyon: or Rod-Fishing in Clear Waters. By Henry Wade, Secretary to the Weardale Angling Association. With Coloured representations of the principal Flies, and other Illustrations. Cr. 8vo. 7*s*. 6*d*.

SERMONS.

PARISH SERMONS. By the Rev. M. F. Sadler, M.A., Vicar of Bridgwater. Author of the "Sacrament of Responsibility," and "The Second Adam and the New Birth." Vol. I. Advent to Trinity. Fcap. 8vo. 7*s*. 6*d*.

Twenty-four Sermons on Christian Doctrine and Practice, and on the Church, By C. J. Blomfield, D.D., late Lord Bishop of London. (*Hitherto unpublished*.) 8vo. 10*s*. 6*d*.

King's College Sermons. By the Rev. E. H. Plumptre, M.A., Divinity Professor. Fcap. 8vo. 2*s*. 6*d*.

Sermons preached in Westminster. By the Rev. C. F. Secretan, M.A., Incumbent of Holy Trinity, Vauxhall-Bridge Road. Fcap. 8vo. 6*s*.

Sermons. By the Rev. A. Gatty, D.D., Vicar of Ecclesfield. 12mo. 8*s*.

Twenty Plain Sermons for Country Congregations and Family Reading. By the Rev. A. Gatty, D.D., Vicar of Ecclesfield. Fcap. 5*s*.

Sermons to a Country Congregation—Advent to Trinity. By the Rev. Hastings Gordon, M.A. 12mo 6*s*.

Gospel Truths in Parochial Sermons for the Great Festivals. By the Rev. J. Townson, M.A. Fcap. 8vo. 2*s*. 6*d*.

The Bible and its Interpreters: being the Substance of Three Sermons preached in the Parish Church, St. Ann, Wandsworth. By James Booth, LL.D., Vicar of Stone, Buckinghamshire. 8vo. 2*s*. 6*d*.

Four Sermons on the "Comfortable Words" in the Office for the Holy Communion. By Alexander Goalen, B.A. Fcap. 8vo. 2*s*.

The Prodigal Son. Sermons by W. R. Clark, M.A., Vicar of Taunton, S. Mary Magdalene. Fcap. 8vo. 2*s*. 6*d*.

Parochial Sermons. By the Rev. D. G. Stacy, Vicar of Hornchurch, Essex. Fcap. 8vo. 5*s*.

Sermons Suggested by the Miracles of our Lord and Saviour Jesus Christ. By the Very Rev. Dean Hook. 2 vols. Fcap. 8vo. 12*s.*

Five Sermons Preached before the University of Oxford. By the Very Rev. W. F. Hook, D.D., Dean of Chichester. *Third Edition.* 8*s.*

Plain Parochial Sermons. By the Rev. C. F. C. Pigott, B.A., late Curate of St. Michael's, Handsworth. Fcap. 8vo. 6*s.*

Our Privileges, Responsibilities, and Trials. By the Rev. E. Phillips, M.A. Fcap. 8vo. 5*s.*

Sermons, chiefly Practical. By the Rev. T. Nunns, M.A. Edited by the Very Rev. W. F. Hook, D.D., Dean of Chichester. Fcap. 8vo. 6*s.*

Sermons, Preached in the Parish Church of Godalming, Surrey, by the Rev. E. J. Boyce, M.A., Vicar. *Second Edition.* Fcap. 8vo. 6*s.*

Life in Christ. By the Rev. J. Llewellyn Davies, M.A., Rector of Christ Church, Marylebone. Fcap. 8vo. 5*s.*

The Church of England; its Constitution, Mission, and Trials. By the Rt. Rev. Bishop Broughton. Edited, with a Prefatory Memoir, by the Ven. Archdeaçon Harrison. 8vo. 10*s.* 6*d.*

Plain Sermons, Addressed to a Country Congregation. By the late E. Blencowe, M.A. 1st and 3rd Series, fcap. 8vo. 7*s.* 6*d.* each.

Occasional Sermons. By a Member of the Church of England. Fcap. 8vo. 2*s.* 6*d.*

Missionary Sermons preached at Hagley. Fcap. 3*s.* 6*d.*

The Sufficiency of Christ. Sermons preached during the Reading Lenten Mission of 1860. Fcap. 8vo. 2*s.* 6*d.*

Westminster Abbey Sermons for the Working Classes. Fcap. *Authorized Edition.* 1858. 2*s.*: 1859. 2*s.* 6*d.*

Sermons preached at St. Paul's Cathedral. *Authorized Edition.* 1859. Fcap. 8vo. 2*s.* 6*d.*

DAILY Readings for a Year, on the Life of Our Lord and Saviour Jesus Christ. By the Rev. Peter Young, M.A. *Second Edition,* improved. 2 vols. Crown 8vo. 1*l.* 1*s.* Antique calf, 1*l.* 16*s.* Morocco, Hayday, 2*l.*

A Commentary on the Gospels for the Sundays and other Holy Days of the Christian Year. By the Rev. W. Denton, A.M., Worcester College, Oxford, and Incumbent of St. Bartholomew's, Cripplegate. 8vo. Vol. 1. Advent to Easter, 15*s.* Vol. II. Easter to the Sixteenth Sunday after Trinity, 14*s.*

Lights of the Morning: or, Meditations for every Day in the Year. From the German of Frederic Arndt. With a Preface by the Rev. W. C. Magee, D.D. Fcap. 8vo. Advent to Whitsuntide, 5*s.* 6*d.* Trinity, 5*s.* 6*d.*

Short Sunday Evening Readings, Selected and Abridged from various Authors by the Dowager Countess of Cawdor. In large type. 8vo. 5*s*.

The Second Adam, and the New Birth; or, the Doctrine of Baptism as contained in Holy Scripture. By the Rev. M. F. Sadler, M.A. Vicar of Bridgewater, Author of "The Sacrament of Responsibility." *Third Edition*, greatly enlarged. Fcap. 8vo. 4*s*. 6*d*.

The Sacrament of Responsibility; or, Testimony of the Scripture to the teaching of the Church on Holy Baptism, with especial reference to the Cases of Infants, and Answers to Objections. *Sixth Edition.* 6*d*.

The Acts and Writings of the Apostles. By C. Pickering Clarke, M.A., late Curate of Teddington. Post 8vo. Vol. I., with Map., 7*s*. 6*d*.

The Spirit of the Hebrew Poetry. By Isaac Taylor, Esq., Author of "The Natural History of Enthusiasm," "Ultimate Civilization," &c. 8vo. 10*s*. 6*d*.

Civilization considered as a Science in Relation to its Essence, its Elements, and its End. By George Harris, F.S.A., of the Middle Temple, Barrister at Law, Author of "The Life of Lord Chancellor Hardwicke." 8vo. 12*s*.

The Wisdom of the Son of David: an Exposition of the First Nine Chapters of the Book of Proverbs. Fcap. 8vo. 5*s*.

A Companion to the Authorized Version of the New Testament: being Explanatory Notes, together with Explanatory Observations and an Introduction. By the Rev. H. B. Hall, B.C.L. *Second and cheaper Edition*, revised and enlarged. Fcap. 8vo. 3*s*. 6*d*.

A History of the Church of England from the Accession of James II. to the Rise of the Bangorian Controversy in 1717. By the Rev. T. Debary, M.A. 8vo. 14*s*.

A Treatise on Metaphysics in Connexion with Revealed Religion. By the Rev. J. H. MacMahon. 8vo. 14*s*.

Aids to Pastoral Visitation, selected and arranged by the Rev. H. B. Browning, M.A., Curate of St. George, Stamford. 8vo. 5*s*.

A Popular Paraphrase of St. Paul's Epistle to the Romans, with Notes. By the Rev. A. C. Bromehead, M.A. Crown 8vo. 3*s*. 6*d*.

Remarks on Certain Offices of the Church of England, popularly termed the Occasional Services. By the Rev. W. J. Dampier. 12mo. 5*s*.

The Sympathy of Christ. Six Readings for the Sundays in Lent, or for the Days of the Holy Week. By the Rev. W. J. Dampier, M.A., Vicar of Coggeshall. *Second Edition.* 18mo. 2*s*. 6*d*.

On Party Spirit in the English Church. By the Rev. S. Robins. 12mo. 2*s*. 6*d*.

Papers on Preaching and Public Speaking. By a Wykehamist. Fcap. 8vo. 5*s.*

This volume is an enlargement and extension, with corrections, of the Papers which appeared in the "Guardian" in 1858-9.

The Speaker at Home. Chapters on Public Speaking and Reading aloud, by the Rev. J. J. Halcombe, M.A., and on the Physiology of Speech, by W. H. Stone, M.A., M.B. *Second Edition.* Fcap. 8vo. 3*s.* 6*d.*

The English Churchman's Signal. By the Writer of "A Plain Word to the Wise in Heart." Fcap. 8vo. 2*s.* 6*d.*

A Plain Word to the Wise in Heart on our Duties at Church, and on our Prayer Book. *Fourth Edition.* Sewed, 1*s.* 6*d.*

Register of Parishioners who have received Holy Confirmation. Arranged by William Fraser, B. C. L., Vicar of Alton. Oblong 4to. 7*s.* 6*d.*; 10*s.* 6*d.*; 12*s.*

Readings on the Morning and Evening Prayer and the Litany. By J. S. Blunt. *Second Edition, enlarged.* Fcap. 8vo. 3*s.* 6*d.*

Confirmation. By J. S. Blunt, Author of "Readings on the Morning and Evening Prayer," &c. Fcap. 8vo. 3*s.* 6*d.*

Life after Confirmation. By the same Author. 18mo. 1*s.*

Welchman on the Thirty-nine Articles of the Church of England, with Scriptural Proofs, &c. 18mo. 2*s.* or interleaved for Students, 3*s.*

Bishop Jewel's Apology for the Church of England, with his famous Epistle on the Council of Trent, and a Memoir. 32mo. 2*s.*

The Book of Psalms (Prayer Book Version). With Short Headings and Explanatory Notes. By the Rev. Ernest Hawkins, B.D., Prebendary of St. Paul's. *Second and cheaper Edition, revised and enlarged,* Fcap. 8vo., cloth limp, red edges, 2*s.* 6*d.*

Family Prayers:—containing Psalms, Lessons, and Prayers, for every Morning and Evening in the Week. By the Rev. Ernest Hawkins, B.D., Prebendary of St. Paul's. *Eighth Edition.* Fcap. 8vo. 1*s.*; sewed, 9*d.*

Household Prayers on Scriptural Subjects, for Four Weeks. With Forms for various occasions. By a Member of the Church of England. *Second Edition, enlarged.* 8vo. 4*s.* 6*d.*

Forms of Prayer adapted to each Day of the Week. For use in Families or Households. By the Rev. John Jebb, D.D., 8vo. 2*s.* 6*d.*

Walton's Lives of Donne, Wotton, Hooker, Herbert, and Sanderson. A New Edition, to which is now added a Memoir of Mr. Isaac Walton, by William Dowling, Esq. of the Inner Temple, Barrister-at-Law. With Illustrative Notes, numerous Portraits, and other Engravings, Index, &c. Crown 8vo. 10*s.* 6*d.* Calf antique, 15*s.* Morocco, 18*s.*

The Life of Martin Luther. By H. Worsley, M.A., Rector of Easton, Suffolk. 2 vols. 8vo. 1*l.* 4*s.*

The Church Hymnal, (with or without Psalms.) 12mo. Large Type, 1*s*. 6*d*. 18mo. 1*s*. 32mo. for Parochial Schools, 6*d*.

This book is now in use in every English Diocese, and is the *Authorized* Book in some of the Colonial Dioceses.

Three Lectures on Archbishop Cranmer. By the Rev. C. J. Burton, M.A., Chancellor of Carlisle. 12mo. 3*s*.

Church Reading: according to the method advised by Thomas Sheridan. By the Rev. J. J. Halcombe, M.A. 8vo. 3*s*. 6*d*.

The Kafir, the Hottentot, and the Frontier Farmer. Passages of Missionary Life from the Journals of the Ven. Archdeacon Merriman. Illustrated. Fcap. 8vo. 3*s*. 6*d*.

Lectures on the Tinnevelly Missions. By the Rev. Dr. Caldwell, of Edeyenkoody. Crown 8vo. 2*s*. 6*d*.

The "Cruise of the Beacon." A Narrative of a Visit to the Islands in Bass's Straits. By the Right Rev. the Bishop of Tasmania. With Illustrations. Crown 8vo. 5*s*.

*** Messrs. Bell and Daldy are agents for all the other Publications of the Society for the Propagation of the Gospel in Foreign Parts.

The Sweet Psalmist of Israel; or, the Life of David, King of Israel; illustrated by his own Psalms, newly versified in various metres. By the Rev. William Shepherd, B.D. Fcap. 8vo. 5*s*.

Giles Witherne; or, The Reward of Disobedience. A Village Tale for the Young. By Raven Witherne. *New Edition, preparing.*

The Disorderly Family; or, the Village of R * * * *. A Tale for Young Persons. In Two Parts. By a Father. 6*d*.; Cloth, gilt edges, 1*s*.

By the Rev. J. Erskine Clarke, *of Derby*.

HEART Music, for the Hearth-Ring; the Street-Walk; the Country Stroll; the Work-Hours; the Rest-Day; the Trouble-Time. *New Edition.* 1*s*. paper; 1*s*. 6*d*. cloth limp.

The Giant's Arrows. A Book for the Children of Working People. 16mo. 6*d*.; cloth, 1*s*.

Children at Church. Twelve Simple Sermons. 2 vols. 1*s*. each; 1*s*. 6*d*. cloth, gilt; or together in 1 vol. cloth gilt, 2*s*. 6*d*.

Little Lectures for Little Folk. 16mo. 1*s*.

Plain Papers on the Social Economy of the People. Fcap. 8vo. 2*s*. 6*d*.

No. 1. Recreations of the People.—No. 2. Penny Banks.—No. 3. Labourers' Clubs and Working Men's Refreshment Rooms.—No. 4. Children of the People. 6*d*. each.

The Devotional Library.

Edited by the Very Rev. W. F. Hook, D.D., Dean of Chichester.

A Series of Works, original or selected from well-known Church of England Divines, published at the lowest price, and suitable, from their practical character and cheapness, for Parochial distribution.

SHORT Meditations for Every Day in the Year. 2 vols. (1260 pages,) 32mo. Cloth, 5*s.*; calf, gilt edges, 9*s.* Calf antique, 12*s.*

In Separate Parts.

ADVENT to LENT, cloth, 1*s.*; limp calf, gilt edges, 2*s.* 6*d.*; LENT, cloth, 9*d.*; calf, 2*s.* 3*d.* EASTER, cloth, 9*d.*; calf, 2*s.* 3*d.* TRINITY, Part I. 1*s.*; calf, 2*s.* 6*d.* TRINITY, Part II. 1*s.*; calf, 2*s.* 6*d.*

⁂ *Large Paper Edition*, 4 vols. fcap. 8vo. large type. 14*s.* Morocco, 30*s.*

The Christian taught by the Church's Services. (490 pages), royal 32mo. Cloth, 2*s.* 6*d.*; calf, gilt edges, 4*s.* 6*d.* Calf antique, 6*s.*

In Separate Parts.

ADVENT TO TRINITY, cloth, 1*s.*; limp calf, gilt edges, 2*s.* 6*d.* TRINITY, cloth, 8*d.*; calf, 2*s.* 2*d.* MINOR FESTIVALS, 8*d.*; calf, 2*s.* 2*d.*

⁂ *Large Paper Edition*, Fcap. 8vo. large type. 6*s.* 6*d.* Calf antique, or morocco, 11*s.* 6*d.*

Devotions for Domestic Use. 32mo. cloth, 2*s.*; calf, gilt edges, 4*s.* Calf antique, 5*s.* 6*d.* Containing:—

- The Common Prayer Book the best Companion in the Family as well as in the Temple. 3*d.*
- Litanies for Domestic Use, 2*d.*
- Family Prayers; or, Morning and Evening Services for every Day in the Week. By the Bishop of Salisbury; cloth, 6*d.*; calf, 2*s.*
- Bishop Hall's Sacred Aphorisms. Selected and arranged with the Texts to which they refer. By the Rev. R. B. Exton, M.A.; cloth, 9*d.*

⁂ These are arranged together as being suitable for Domestic Use; but they may be had separately at the prices affixed.

Aids to a Holy Life. First Series. 32mo. Cloth, 1*s.* 6*d.*; calf, gilt edges, 3*s.* 6*d.* Calf antique, 5*s.* Containing:—

- Prayers for the Young. By Dr. Hook, ½*d.*
- Pastoral Address to a Young Communicant. By Dr. Hook, ½*d.*
- Helps to Self-Examination. By W. F. Hook, D.D., ½*d.*
- Directions for Spending One Day Well. By Archbishop Synge, ½*d.*
- Rules for the Conduct of Human Life. By Archbishop Synge. 1*d.*
- The Sum of Christianity, wherein a short and plain Account is given of the Christian Faith; Christian's Duty; Christian Prayer; Christian Sacrament. By C. Ellis, 1*d.*
- Ejaculatory Prayer; or, the Duty of Offering up Short Prayers to God on all Occasions. By R. Cook. 2*d.*
- Prayers for a Week. From J. Sorocold, 2*d.*
- Companion to the Altar; being Prayers, Thanksgivings, and Meditations. Edited by Dr. Hook. Cloth, 6*d.*

⁂ Any of the above may be had for distribution at the prices affixed; they are arranged together as being suitable for Young Persons and for Private Devotion.

The Devotional Library continued.

Aids to a Holy Life. Second Series. 32mo. Cloth, 2*s.*; calf, gilt edges, 4*s.* Calf antique, 5*s.* 6*d.* Containing:—

Holy Thoughts and Prayers, arranged for Daily Use on each Day in the Week, 3*d.*

The Retired Christian exercised on Divine Thoughts and Heavenly Meditations. By Bishop Ken. 3*d.*

Penitential Reflections for the Holy Season of Lent, and other Days of Fasting and Abstinence during the Year. 6*d.*

The Crucified Jesus; a Devotional Commentary on the XXII and XXIII Chapters of St. Luke. By A. Horneck, D.D. 3*d.*

Short Reflections for every Morning and Evening during the Week. By N. Spinckes, 2*d.*

The Sick Man Visited; or, Meditations and Prayers for the Sick Room. By N. Spinckes, 3*d.*

*** These are arranged together as being suitable for Private Meditation and Prayer: they may be had separately at the prices affixed.

Helps to Daily Devotion. 32mo. Cloth, 8*d.* Containing:—

The Sum of Christianity, 1*d.*
Directions for spending One Day Well, ½*d.*
Helps to Self-Examination, ½*d.*
Short Reflections for Morning and Evening, 2*d.*
Prayers for a Week, 2*d.*

The History of our Lord and Saviour Jesus Christ; in Three Parts, with suitable Meditations and Prayers. By W. Reading, M.A. 32mo. Cloth, 2*s.*; calf, gilt edges, 4*s.* Calf antique, 5*s.* 6*d.*

Hall's Sacred Aphorisms. Selected and arranged with the Texts to which they refer, by the Rev. R. B. Exton, M.A. 32mo. cloth, 9*d.*; limp calf, gilt edges, 2*s.* 3*d.*

Devout Musings on the Book of Psalms. 2 vols. 32mo. Cloth, 5*s.*; calf, gilt edges, 9*s.*; calf antique, 12*s.* Or, in four parts, price 1*s.* each; limp calf, gilt edges, 2*s.* 6*d.*

The Church Sunday School Hymn Book. 32mo. cloth, 8*d.*; calf, gilt edges, 2*s.* 6*d.*

*** A *Large Paper Edition* for Prizes, &c. 1*s.* 6*d.*; calf, gilt edges, 3*s.* 6*d.*

SHORT Meditations for Every Day in the Year. Edited by the Very Rev. W. F. Hook, D.D. *New Edition.* 4 vols. fcap. 8vo., large type, 14*s.*; morocco, 30*s.*

The Christian taught by the Church's Services. Edited by the Very Rev. W. F. Hook, D. D. *New Edition*, fcap. 8vo. large type, 6*s.* 6*d.* Antique calf, or morocco, 11*s.* 6*d.*

Holy Thoughts and Prayers, arranged for Daily Use on each Day of the Week, according to the stated Hours of Prayer. *Fifth Edition*, with additions. 16mo. Cloth, red edges, 2*s.*; calf, gilt edges, 3*s.*

A Companion to the Altar. Being Prayers, Thanksgivings, and Meditations, and the Office of the Holy Communion. Edited by the Very Rev. W. F. Hook, D. D. *Second Edition.* Handsomely printed in red and black. 32mo. Cloth, red edges, 2*s.* Morocco, 3*s.* 6*d.*

The Church Sunday School Hymn Book. Edited by W. F. Hook, D.D. *Large paper.* Cloth, 1*s.* 6*d.*; calf, gilt edges, 3*s.* 6*d.*

*** For cheap editions of the above Five Books, see List of the Devotional Library.

EDUCATIONAL BOOKS.

Bibliotheca Classica.

A Series of Greek and Latin Authors. With English Notes. 8vo. Edited by various Scholars, under the direction of G. Long, Esq., M.A., Classical Lecturer of Brighton College: and the late Rev. A. J. Macleane, M.A., Head Master of King Edward's School, Bath.

AESCHYLUS. By F. A. Paley, M.A. 18*s*.

Cicero's Orations. Edited by G. Long, M.A. 4 vols. Vol. I. 16*s*.; Vol. II. 14*s*; Vol. III. 16*s*.; Vol. IV. 18*s*.

Demosthenes. By R. Whiston, M.A., Head Master of Rochester Grammar School. Vol. I. 16*s*. Vol. II. *preparing*.

Euripides. By F. A. Paley, M.A. 3 vols. 16*s*. each.

Herodotus. By J. W. Blakesley, B.D., late Fellow and Tutor of Trinity College, Cambridge. 2 vols. 32*s*.

Hesiod. By F. A. Paley, M.A. 10*s*. 6*d*.

Horace. By A. J. Macleane, M.A. 18*s*.

Juvenal and Persius. By A. J. Macleane, M.A. 14*s*.

Sophocles. By F. H. Blaydes, M.A. Vol. I. 18*s*. Vol. II. *preparing*.

Terence. By E. St. J. Parry, M.A., Balliol College, Oxford. 18*s*.

Virgil. By J. Conington, M.A., Professor of Latin at Oxford. Vol. I. containing the Bucolics and Georgics. 12*s*. Vol. II. *in the press*.

Plato. By W. H. Thompson, M.A. Vol. I. [*Preparing*.

Grammar-School Classics.

A Series of Greek and Latin Authors. Newly Edited, with English Notes for Schools. Fcap. 8vo.

CAESARIS Commentarii de Bello Gallico. *Second Edition*. By G. Long, M.A. 5*s*. 6*d*.

Caesar de Bello Gallico, Books 1 to 3. With English Notes for Junior Classes. By G. Long, M.A. 2*s*. 6*d*.

M. Tullii Ciceronis Cato Major, Sive de Senectute, Laelius, Sive de Amicitia, et Epistolae Selectae. By G. Long, M.A. 4*s*. 6*d*.

Quinti Horatii Flacci Opera Omnia. By A. J. Macleane, 6*s*. 6*d*.

Grammar-School Classics continued.

P. Ovidii Nasonis Fastorum Libri Sex. By F. A. Paley. 5*s*.

C. Sallustii Crispi Catilina et Jugurtha. By G. Long, M.A. 5*s*.

Taciti Germania et Agricola. By P. Frost, M.A. 3*s*. 6*d*.

Xenophontis Anabasis, with Introduction; Geographical and other Notes, Itinerary, and Three Maps compiled from recent surveys. By J. F. Macmichael, B.A. *New Edition*. 5*s*.

Xenophontis Cyropaedia. By G. M. Gorham, M.A., late Fellow of Trinity College, Cambridge. 6*s*.

Uniform with the above.

The New Testament in Greek. With English Notes and Prefaces by J. F. Macmichael, B.A. 730 pages. 7*s*. 6*d*.

Cambridge Greek and Latin Texts.

This series is intended to supply for the use of Schools and Students cheap and accurate editions of the Classics, which shall be superior in mechanical execution to the small German editions now current in this country, and more convenient in form.

The texts of the *Bibliotheca Classica* and *Grammar School Classics*, so far as they have been published, will be adopted. These editions have taken their place amongst scholars as valuable contributions to the Classical Literature of this country, and are admitted to be good examples of the judicious and practical nature of English scholarship; and as the editors have formed their texts from a careful examination of the best editions extant, it is believed that no texts better for general use can be found.

The volumes will be well printed at the Cambridge University Press, in a 16mo. size, and will be issued at short intervals.

AESCHYLUS, ex novissima recensione F. A. Paley. 3*s*.

Cæsar de Bello Gallico, ex recensione G. Long, A.M. 2*s*.

Cicero, de Senectute et de Amicitia et Epistolæ Selectæ, ex recensione G. Long, A.M. 1*s*. 6*d*.

Euripides, ex recensione F. A. Paley, A. M. 3 vols. 3*s*. 6*d*. each.

Herodotus, ex recensione J. W. Blakesley, S.T.B. 2 vols. 7*s*.

Horatius, ex recensione A. J. Macleane, A.M. 2*s*. 6*d*.

Lucretius, recognovit H. A. J. Munro, A.M. 2*s*. 6*d*.

Thucydides, ex recensione J. G. Donaldson, S.T.P. 2 vols. 7*s*.

Vergilius, ex recensione J. Conington, A.M. 3*s*. 6*d*.

Xenophontis Anabasis, ex recensione J. F. Macmichael, B.A. [*Shortly*.

Novum Testamentum Graecum Textus Stephanici, 1550. Accedunt variae Lectiones editionum Bezae, Elzeviri, Lachmanni, Tischendorfii, Tregellesii, curante F. H. Scrivener, A.M. 4*s*. 6*d*.

Also, on 4to. writing paper, for MSS. notes. Half bound, gilt top, 12*s*.

Foreign Classics.

With English Notes for Schools. Uniform with the Grammar School Classics. Fcap. 8vo.

AVENTURES de Télémaque, par Fénelon. Edited by C. J. Delille. *Second Edition, revised.* 4s. 6d.

Histoire de Charles XII. par Voltaire. Edited by L. Direy. *Second Edition, revised.* 3s. 6d.

Select Fables of La Fontaine. *Second Edition, revised.* Edited by F. Gasc, M.A. 3s.

"None need now be afraid to introduce this eminently French author, either on account of the difficulty of translating him, or the occasional licence of thought and expression in which he indulges. The renderings of idiomatic passages are unusually good, and the purity of English perfect."—*Athenæum.*

Picciola, by X. B. Saintine. Edited by Dr. Dubuc. 3s. 6d.

This interesting story has been selected with the intention of providing for schools and young persons a good specimen of contemporary French literature, free from the solecisms which are frequently met with in writers of a past age.

The Wallenstein of Schiller, with Notes by Dr. A. Buchheim. [*Immediately.*

Classical Tables. 8vo.

NOTABILIA Quædam: or, the principal tenses of such Irregular Greek Verbs and such elementary Greek, Latin, and French Constructions as are of constant occurrence. 1s. 6d.

Greek Accidence. By the Rev. P. Frost, M. A. 1s.

Latin Accidence. By the Rev. P. Frost, M. A. 1s.

Latin Versification. 1s.

The Principles of Latin Syntax. 1s.

Homeric Dialect: its leading Forms and Peculiarities. By J. S. Baird, T.C.D. 1s. 6d.

A Catalogue of Greek Verbs, Irregular and Defective; their leading formations, tenses in use, and dialectic inflexions; with a copious Appendix, containing Paradigms for conjugation, Rules for formation of tenses, &c. &c. By J. S. Baird, T.C.D. *New Edition, revised.* 3s. 6d.

Richmond Rules to form the Ovidian Distich, &c. By J. Tate, M.A. *New Edition, revised.* 1s. 6d.

AN Atlas of Classical Geography, containing 24 Maps; constructed by W. Hughes, and edited by G. Long. *New Edition*, with coloured outlines, and an Index of Places. 12s. 6d.

A Grammar School Atlas of Classical Geography. The Maps constructed by W. Hughes, and edited by G. Long. Imp. 8vo. 5s.

First Classical Maps, with Chronological Tables of Grecian and Roman History, Tables of Jewish Chronology, and a Map of Palestine. By the Rev. J. Tate, M.A. *Third Edition.* Imp. 8vo. 7s. 6d.

The Choephorae of Æschylus and its Scholia. Revised and interpreted by J. F. Davies, Esq., B.A., Trinity College, Dublin. 8vo. [*In the Press.*

A Latin Grammar. By T. Hewitt Key, M.A., F.R S., Professor of Comparative Grammar, and Head Master of the Junior School, in University College. *Third Edition, revised.* Post 8vo. 8*s.*

A Short Latin Grammar, for Schools. By T. H. Key, M.A., F.R.S. *Third Edition.* Post 8vo. 3*s.* 6*d.*

Latin Accidence. Consisting of the Forms, and intended to prepare boys for Key's Short Latin Grammar. Post 8vo. 2*s.*

Rules for the Genders of Latin Nouns, and the Perfects and Supines of Verbs; with hints on Construing, &c. By H. Haines, M.A. 1*s.* 6*d.*

Materials for Latin Prose Composition. By the Rev. P. Frost, M.A., St. John's College, Cambridge. *Second Edition.* 12mo. 2*s.* 6*d.* Key, 4*s.*

The Works of Virgil, closely rendered into English Rhythm, and illustrated from British Poets of the 16th, 17th, and 18th Centuries. By the Rev. R. C. Singleton, M.A. 2 vols. post 8vo. 18*s.*

Quintus Horatius Flaccus. Illustrated with 50 Engravings from the Antique. Fcap. 8vo. 5*s.* Morocco, 9*s.*

Selections from Ovid: Amores, Tristia, Heroides, Metamorphoses. With English Notes, by the Rev. A. J. Macleane, M.A. Fcap. 8vo. 3*s.* 6*d.*

Sabrinae Corolla in hortulis Regiae Scholae Salopiensis contexuerunt tres viri floribus legendis. *Editio Altera.* 8vo. 12*s.* Morocco, 21*s.*

A First Cheque Book for Latin Verse Makers. By the Rev. F. Gretton, Stamford Free Grammar School. 1*s.* 6*d.* Key, 2*s.* 6*d.*

This little volume contains about 60 sets of verses, graduated in difficulty:—the exercise is to be torn out by the Master, and the paper is to be handed up by the boy when he has written the Latin Version underneath.

Reddenda; or Passages with Parallel Hints for translation into Latin Prose and Verse. By the Rev. F. E. Gretton. Crown 8vo. 4*s.* 6*d.*

Auxilia Graeca: containing Forms of Parsing and Greek Trees, the Greek Prepositions, Rules of Accentuation, Greek Idioms, &c. &c. By the Rev. H. Fowler, M.A. 12mo. 3*s.* 6*d.*

Rudimentary Art Instruction for Artisans and others, and for Schools. FREEHAND OUTLINE. Part I. OUTLINE FROM OUTLINE, or from the Flat. 3*s.* Part II. OUTLINE FROM OBJECTS, or from the Round. 4*s.* By John Bell, Sculptor. Oblong 4to.

A Graduated Series of Exercises in Elementary Algebra, with an Appendix containing Papers of Miscellaneous Examples. Designed for the Use of Schools. By the Rev. G. F. Wright, M.A., Mathematical Master at Wellington College. Crown 8vo. 3*s.* 6*d.*

The Elements of Euclid. Books I.—VI. XI. 1—21; XII. 1, 2; a new text, based on that of Simson, with Exercises. Edited by H. J. Hose, late Mathematical Master of Westminster School. Fcap 4*s.* 6*d.*

A Graduated Series of Exercises on the Elements of Euclid: Books I.—VI.; XI. 1—21; XII. 1, 2. Selected and arranged by Henry J. Hose, M.A. 12mo. 1*s.*

The Enunciations and Figures belonging to the Propositions in the First Six and part of the Eleventh Books of Euclid's Elements, (usually read in the Universities,) prepared for Students in Geometry. By the Rev. J. Brasse, D.D. *New Edition.* Fcap. 8vo. 1*s.* On cards, in case, 5*s.* 6*d.*; without the Figures, 6*d.*

A Table of Anti-Logarithms; containing to seven places of deci-mals, natural numbers, answering to all Logarithms from ·00001 to ·99999; and an improved table of Gauss' Logarithms, by which may be found the Logarithm of the sum or difference of two quantities. With an Appendix, containing a Table of Annuities for three Joint Lives at 3 per cent. Carlisle. By H. E. Filipowski. *Third Edition.* 8vo. 15*s.*

Handbook of the Slide Rule: showing its applicability to Arith-metic, including Interest and Annuities; Mensuration, including Land Surveying. With numerous Examples and useful Tables. By W. H. Bayley, H. M. East India Civil Service. 12mo. 6*s.*

The Mechanics of Construction; including the Theories on the Strength of Materials, Roofs, Arches, and Suspension Bridges. With numerous Examples. By Stephen Fenwick, Esq., of the Royal Military Academy, Woolwich. 8vo. 12*s.*

A New French Course, by Mons. F. E. A. Gasc, M.A.

French Master at Brighton College.

Le Petit Compagnon: a French Talk-book for Little Children. With numerous woodcuts. 16mo. 3*s.* 6*d.* [*Shortly.*

First French Book; being a New, Practical, and Easy Method of Learning the Elements of the French Language. *New Edition.* Fcap. 8vo. 1*s.* 6*d.*

French Fables, for Beginners, in Prose, with an Index of all the words at the end of the work. Fcap. 8vo. 2*s.*

Second French Book; being a Grammar and Exercise Book, on a new and practical plan, exhibiting the chief peculiarities of the French Language, as compared with the English, and intended as a sequel to the "First French Book." Fcap. 8vo. 2*s.* 6*d.*

A Key to the First and Second French Books. Fcap. 8vo. 3*s.* 6*d.*

Histoires Amusantes et Instructives; or, Selections of Complete Stories from the best French Authors, who have written for the Young. With English Notes. *New Edition.* Fcap. 8vo. 2*s.* 6*d.*

Practical Guide to Modern French Conversation: containing:— I. The most current and useful Phrases in Every-Day Talk; II. Everybody's Necessary Questions and Answers in Travel-Talk. Fcap. 2*s.* 6*d.*

French Poetry for the Young. With English Notes, and pre-ceded by a few plain Rules of French Prosody. Fcap. 8vo. 2*s.*

Materials for French Prose Composition; or, Selections from the best English Prose Writers. With copious Foot Notes, and Hints for Idiomatic Renderings. *New Edition.* Fcap. 8vo. 4*s.* 6*d.* Key, 6*s.*

THE French Drama; being a Selection of the best Tragedies and Comedies of Molière, Racine, P. Corneille, T. Corneille, and Voltaire. With Arguments in English at the head of each scene, and Notes, Critical and Explanatory, by A. Gombert. 18mo. Sold separately at 1*s*. each. Half-bound, 1*s*. 6*d*. each.

COMEDIES BY MOLIERE.

Le Misanthrope.
L'Avare.
Le Bourgeois Gentilhomme.
Le Tartuffe.
Le Malade Imaginaire.
Les Femmes Savantes.
Les Fourberies de Scapin.
Les Précieuses Ridicules.
L'Ecole des Femmes.
L'Ecole des Maris.
Le Médecin Malgré Lui.
M. de Pouceaugnac.
Amphitryon.

TRAGEDIES, &c. BY RACINE.

La Thébaïde, ou les Frères Ennemis.
Alexandre le Grand.
Andromaque.
Les Plaideurs, (*Com.*)
Britannicus.
Bérénice.
Bajazet.
Mithridate.
Iphigénie.
Phédre.
Esther.
Athalie.

TRAGEDIES, &c. BY P. CORNEILLE.

Le Cid.
Horace.
Cinna.
Polyeucte.
Pompée.

BY T. CORNEILLE.

Ariane.

PLAYS BY VOLTAIRE.

Brutus.
Zaire.
Alzire.
Orestes.
Le Fanatisme.
Mérope.
La Mort de César.
Semiramis.

Le Nouveau Trésor: or, French Student's Companion: designed to facilitate the Translation of English into French at Sight. *Thirteenth Edition*, with Additions. By M. E*** S*****. 12mo. Roan, 3*s*. 6*d*.

A Test-Book for Students: Examination Papers for Students preparing for the Universities or for Appointments in the Army and Civil Service, and arranged for General Use in Schools. By the Rev. Thomas Stantial, M.A., Head Master of the Grammar School, Bridgwater. Part I.—History and Geography. 2*s*. 6*d*. Part II.—Language and Literature. 2*s*. 6*d*. Part III.—Mathematical Science. 2*s*. 6*d*. Part IV.—Physical Science. 1*s*. 6*d*. Or in 1 vol., Crown 8vo., 7*s*. 6*d*.

Tables of Comparative Chronology, illustrating the division of Universal History into Ancient, Mediæval, and Modern History; and containing a System of Combinations, distinguished by a particular type, to assist the Memory in retaining Dates. By W. E. Bickmore and the Rev. C. Bickmore, M.A. *Third Edition*. 4to. 5*s*.

A Course of Historical and Chronological Instruction. By W. E. Bickmore. 2 Parts. 12mo. 3*s*. 6*d*. each.

A Practical Synopsis of English History: or, A General Summary of Dates and Events for the use of Schools, and Candidates for Public Examinations. By Arthur Bowes. *Third Edition*, enlarged. 8vo. 2*s*.

The Student's Text-Book of English and General History, from B. C. 100 to the present time. With Genealogical Tables, and a Sketch of the English Constitution. By D. Beale. *Sixth Edition.* Post 8vo. Sewed, 2*s.* Cloth, 2*s.* 6*d.*

"This is very much in advance of most works we have seen devoted to similar purposes. We can award very high praise to a volume which may prove invaluable to teachers and taught."—*Athenæum.*

The Elements of the English Language for Schools and Colleges. By Ernest Adams, Ph. D. University College School. *New Edition, enlarged.* [*In the Press.*

The Geographical Text-Book; a Practical Geography, calculated to facilitate the study of that useful science, by a constant reference to the Blank Maps. By M. E . . . S . . . . . 12mo. 2*s.*

II. The Blank Maps done up separately. 4to. 2*s.* coloured.

Under Government: an Official Key to the Civil Service, and Guide for Candidates seeking Appointments under the Crown. By Joseph Charles Parkinson, Inland Revenue, Somerset House. *New Edition.* [*Preparing.*

Government Examinations; being a Companion to "Under Government," and a Guide to the Civil Service Examinations. By J. C. Parkinson. Crown 8vo. 2*s.* 6*d.*

The Manual of Book-keeping; by an Experienced Clerk. 12mo. *Eighth Edition.* 4*s.*

Double Entry Elucidated. By B. W. Foster. 4to. 8*s.* 6*d.*

Penmanship, Theoretical and Practical, Illustrated and Explained. By B. F. Foster. 12mo. *New Edition.* 2*s.* 6*d.*

Goldsmith's (J.) Copy Books: five sorts, large, text, round, small, and mixed. Post 4to. on fine paper. 6*s.* per dozen.

The Young Ladies' School Record: or, Register of Studies and conduct. 12mo. 6*d.*

A Short Explanation of the Epistles and Gospels of the Christian Year, with Questions for Schools. Royal 32mo. 2*s.* 6*d.*; calf, 4*s.* 6*d.*

Manual of Astronomy: a Popular Treatise on Descriptive, Phy-sical, and Practical Astronomy. By John Drew, F.R.A.S. *Second Edition.* Fcap. 8vo. 5*s.*

The First Book of Botany. Being a Plain and Brief Introduction to that Science for Schools and Young Persons. By Mrs. Loudon. Illustrated with 36 Wood Engravings. *Second Edition.* 18mo. 1*s.*

A Handy Book of the Chemistry of Soils. By John Scoffern. M. B. Crown 8vo. [*In the Press.*

English Poetry for Classical Schools; or, Florilegium Poeticum Anglicanum. 12mo. 1*s.* 6*d.*

Hints and Helps for Youths leaving School. By the Rev. J. S. Gilderdale, M.A. Fcap. 8vo. 5*s.* Calf, 8*s.* 6*d.*

BELL AND DALDY'S ILLUSTRATED SCHOOL BOOKS.

Royal 16mo.

SCHOOL Primer. 6*d.*

School Reader. 1*s.* [*Shortly.*

Poetry Book for Schools. 1*s.*

COURSE OF INSTRUCTION FOR THE YOUNG, BY HORACE GRANT.

EXERCISES for the Improvement of the Senses; for Young Childdren. 18mo. 1*s.* 6*d.*

Geography for Young Children. *New Edition.* 18mo. 2*s.*

Arithmetic for Young Children. *New Edition.* 18mo. 1*s.* 6*d.*

Arithmetic. Second Stage. *New Edition.* 18mo. 3*s.*

PERIODICALS.

NOTES and Queries: a Medium of Intercommunication for Literary Men, Artists, Antiquaries, Genealogists, &c. Published every Saturday. 4to. 4*d.*, stamped, 5*d.* Vols. I. to XII. Second Series now ready, 10*s.* 6*d.* each.
*** General Index to the First Series, 5*s.*
——————— Second Series, 5*s.*

The Monthly Medley for Happy Homes. A New Miscellany for Children: Conducted by the Rev. J. Erskine Clarke. Price 1*d.* Volumes for 1860 and 1861, 1*s.* 6*d.* each.

The Parish Magazine. Edited by J. Erskine Clarke, M.A., Derby. Monthly, price 1*d.* Volumes for 1859, 1860, and 1861, 1*s.* 6*d.* and 2*s.* each.

The Mission Field: a Monthly Record of the Proceedings of the Society for the Propagation of the Gospel. Vols. II. to VI. post 8vo. 3*s.* each. (Vol. I. is out of print.) Continued in Numbers, 2*d.* each.

The Gospel Missionary. Published for the Society for the Propagation of the Gospel in Foreign Parts, Monthly at ½*d.* Vols. II. to XI. in cloth, 1*s.* each. (Vol. I. is out of print.)

Missions to the Heathen; being Records of the Progress of the Efforts made by the Society for the Propagation of the Gospel in Foreign Parts for the Conversion of the Heathen. Published occasionally in a cheap form for distribution, at prices varying from 1*d.* to 1*s.* 6*d.* each. Nos. 1 to 43 are already published.

Church in the Colonies, consisting chiefly of Journals by the Colonial Bishops of their Progress and Special Visitations. Published occasionally at prices varying from 2*d.* to 1*s.* 6*d.* each. Nos. 1 to 37 are already published.

CLARKE'S COPY-BOOKS. Price 3*d*. Extra Size, on Fine Paper, 6*d*.

The FIRST COPY-BOOK contains *elementary turns*, with a broad mark like a T, which divides a well-formed turn into two equal parts. This exercise enables the learner to judge of *form, distance, and proportion*.

The SECOND contains *large-hand letters*, and the means by which such letters may be properly combined; the joinings in writing being probably as difficult to learn as the form of each character. This book also gives the whole alphabet, not in separate letters, but rather as one *word*; and, at the end of the alphabet, the difficult letters are repeated so as to render the writing of the pupil more thorough and *uniform*.

The THIRD contains *large-hand words*, commencing with *unflourished* capitals; and the words being short, the capitals in question receive the attention they demand. As Large, and Extra Large-text, to which the fingers of the learner are not equal, have been dispensed with in this series, the popular objection of having *too many Copy-books* for the pupil to drudge through, is now fairly met. When letters are very large, the scholar cannot compass them without stopping to change the position of his hand, which *destroys* the *freedom* which such writing is intended to promote.

The FOURTH contains the essentials of a useful kind of *small-hand*. There are first, as in large-hand, five easy letters of the alphabet, forming four copies, which of course are repeated. Then follows the remainder of the alphabet, with the difficult characters alluded to. The letters in this hand, especially the *a*, *c*, *d*, *g*, *o*, and *q*, are so formed that when the learner will have to correspond, his writing will not appear stiff. The copies in this book are not *mere Large-hand reduced*.

The FIFTH contains *small-hand copies*, with instructions as to the manner in which the pupil should hold his pen, so that when he leaves school he may not merely have some facility in copying, but really possess the information on the subject of writing which he may need at any future time.

The SIXTH contains the foundation for a style of *small-hand*, adapted to females, *moderately pointed*.

The SEVENTH contains copies for females; and the holding of the pen is, of course, the subject to which they specially relate.

This Series is specially adapted for those who are preparing for a commercial life. It is generally found when a boy leaves school that his writing is of such a character that it is some months before it is available for book-keeping or accounts. The special object of this Series of Copy-Books is to form his writing in such a style that he may be put to the work of a counting-house at once. By following this course from the first the writing is kept free and legible, whilst it avoids unnecessary flourishing.

Specimens of hand-writing after a short course may be seen on application to the Publishers.

BELL AND DALDY'S

POCKET VOLUMES

A SERIES OF SELECT WORKS OF

FAVOURITE AUTHORS.

THE intention of the Publishers is to produce a Serie Volumes adapted for general reading, moderate in p compact and elegant in form, and executed in a s.yl fitting them to be permanently preserved.

They do not profess to compete with the so-called cheap volumes. They believe that a cheapness which is attained by the use of inferior type and paper, and absence of editorial care, and which results in volumes that no one cares to keep, is a false cheapness. They desire rather to produce books superior in quality, and relatively as cheap.

Each volume will be carefully revised by a competent editor, and printed at the Chiswick Press, on fine paper, with new type, and ornaments and initial letters specially designed for the series.

The *Pocket Volumes* will include all classes of Literature, both copyright and non-copyright;—Biography, History, Voyages, Travels, Poetry, sacred and secular, Books of Adventure and Fiction. They will include Translations of Foreign Books, and also such American Literature as may be considered worthy of adoption.

The Publishers desire to respect the moral claims of authors who cannot secure legal copyright in this country, and to remunerate equitably those whose works they may reprint.

The books will be issued at short intervals, in paper covers, at various prices, from 1*s.* to 3*s.* 6*d.*, and in cloth, top edge gilt, at 6*d.* per volume extra, in half morocco, Roxburgh style, at 1*s.* extra, in antique or best plain morocco (Hayday), at 4*s.* extra.

Now Ready.

The Midshipman. By Capt. Basil Hall, R.N. 3*s.*
Southey's Life of Nelson. 2*s.* 6*d.*
George Herbert's Poems. 2*s.*
George Herbert's Works. 3*s.*
Longfellow's Poems. 2*s.* 6*d.*
Lamb's Tales from Shakspeare. 2*s.* 6*d.*
Milton's Paradise Lost. 2*s.* 6*d.*
Milton's Paradise Regained and other Poems. 2*s.* 6*d.*

Preparing.

The Robin Hood Ballads.
The Lieutenant and Commander. By Capt. Basil Hall, R.N.
Sea Songs and Ballads. By Charles Dibdin, and others.
Walton's Lives of Donne, Wotton, Hooker, &c.
Walton's Complete Angler.

Other Works are in preparation.

BOUND BY
BONE & SON,
76, FLEET STREET,
LONDON.